

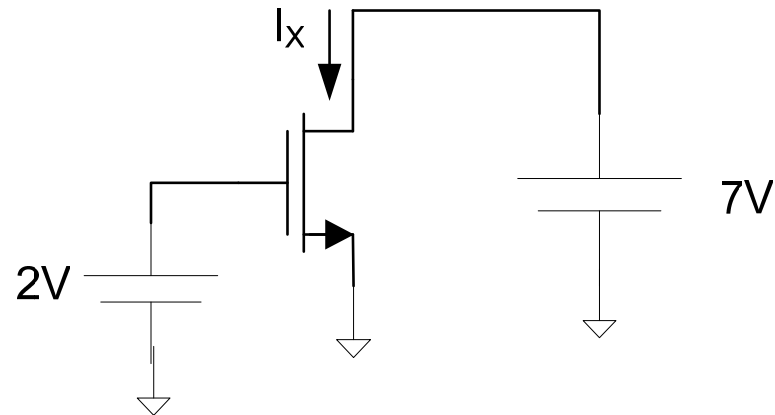
EE 230
Lecture 31

THE MOS TRANSISTOR

Model Simplifications

THE Bipolar Junction TRANSISTOR

Quiz 31



Determine I_x . Assume $W=100\mu$, $L=2\mu$, $V_T=1V$, $\mu C_{OX}=10^{-4}A/V^2$, $\lambda=0$

And the number is ?

1

3

8

5

4

?

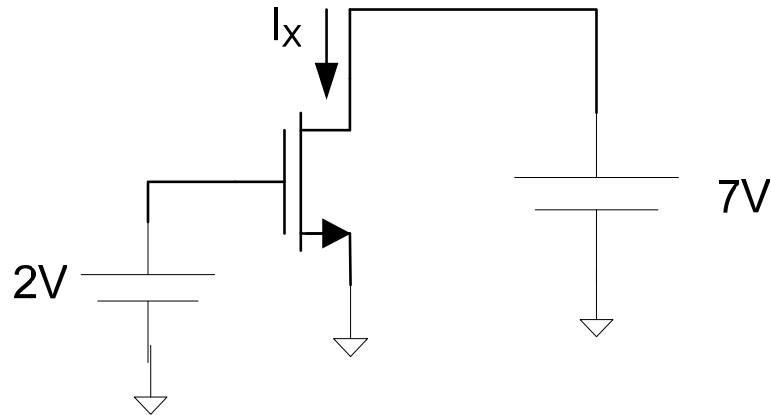
2

6

9

7

Quiz 31 (solution)



Determine I_x . Assume $W=100\mu$, $L=2\mu$, $V_T=1V$, $\mu C_{OX}=10^{-4}A/V^2$, $\lambda=0$

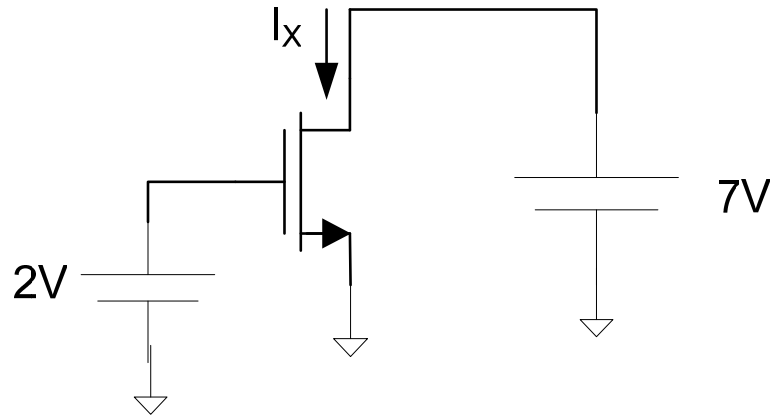
$$I_G = 0$$

$$I_D = \begin{cases} 0 & V_{GS} \leq V_T & \leftarrow \text{Cutoff} \\ \mu C_{OX} \frac{W}{L} \left(V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_T, V_{DS} < V_{GS} - V_T & \leftarrow \text{Triode} \\ \mu C_{OX} \frac{W}{2L} (V_{GS} - V_T)^2 \cdot (1 + \lambda V_{DS}) & V_{GS} \geq V_T, V_{DS} \geq V_{GS} - V_T & \leftarrow \text{Saturation} \end{cases}$$

Guess Saturation:

$$I_D = \mu C_{OX} \frac{W}{2L} (V_{GS} - V_T)^2 \quad V_{GS} \geq V_T \quad V_{DS} > V_{GS} - V_T$$

Quiz 31 (solution)



Determine I_x . Assume $W=100\mu$, $L=2\mu$, $V_T=1V$, $\mu C_{OX}=10^{-4}A/V^2$, $\lambda=0$

Guess Saturation:

$$I_D = \mu C_{ox} \frac{W}{2L} (V_{GS} - V_T)^2$$

$$V_{GS} \geq V_T$$

$$V_{DS} > V_{GS} - V_T$$

$$I_D = 10^{-4} \frac{100\mu}{2 \cdot 2\mu} (2-1)^2$$

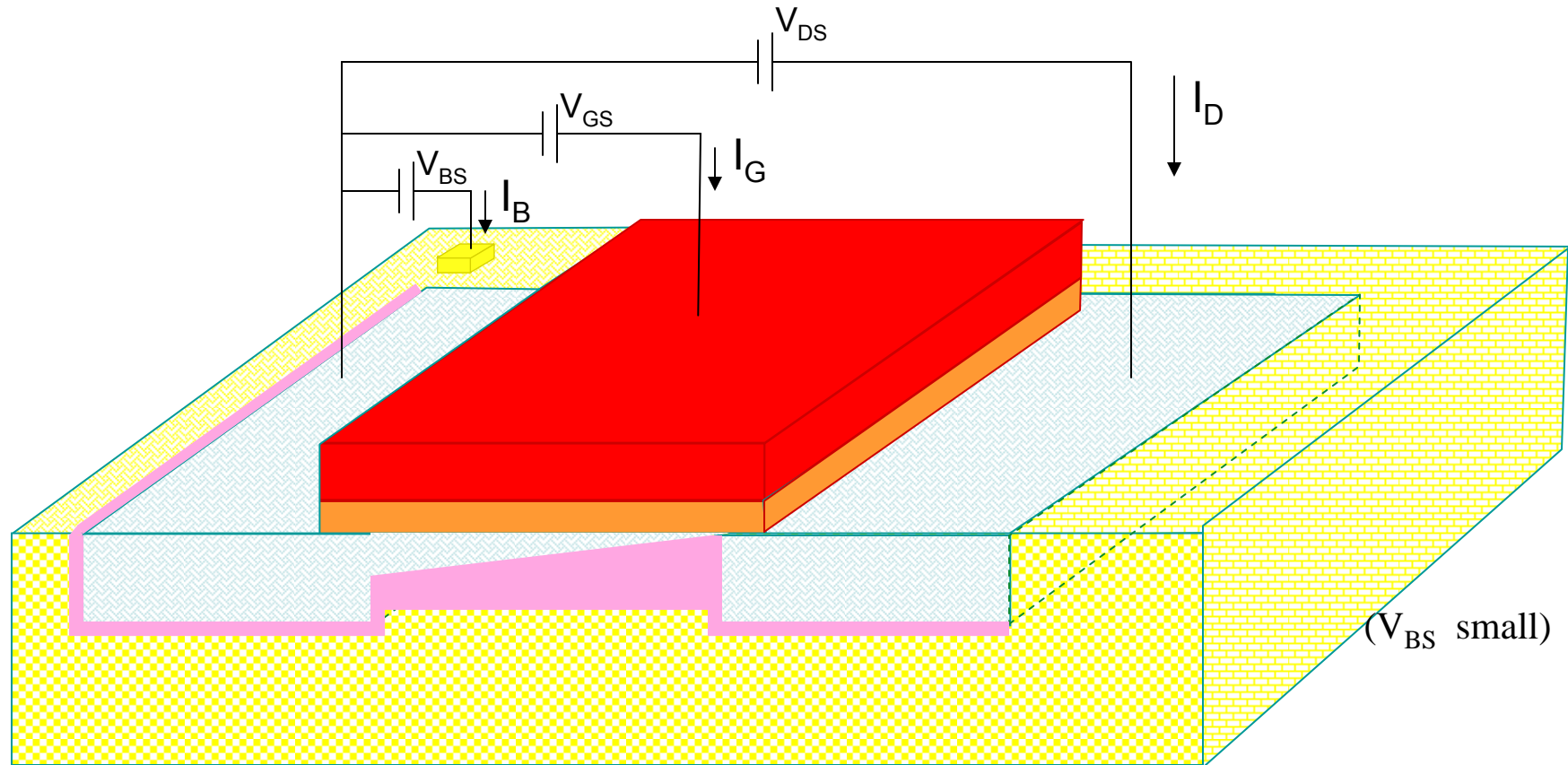
$$2V \geq 1V$$

$$7V > 2V - 1V$$

$$I_D = 2.5mA$$

Review from Last Time:

n-Channel MOSFET Operation and Model



“Saturation” region of operation

Inversion layer disappears near drain

Saturation first occurs when $V_{DS} = V_{GS} - V_T$

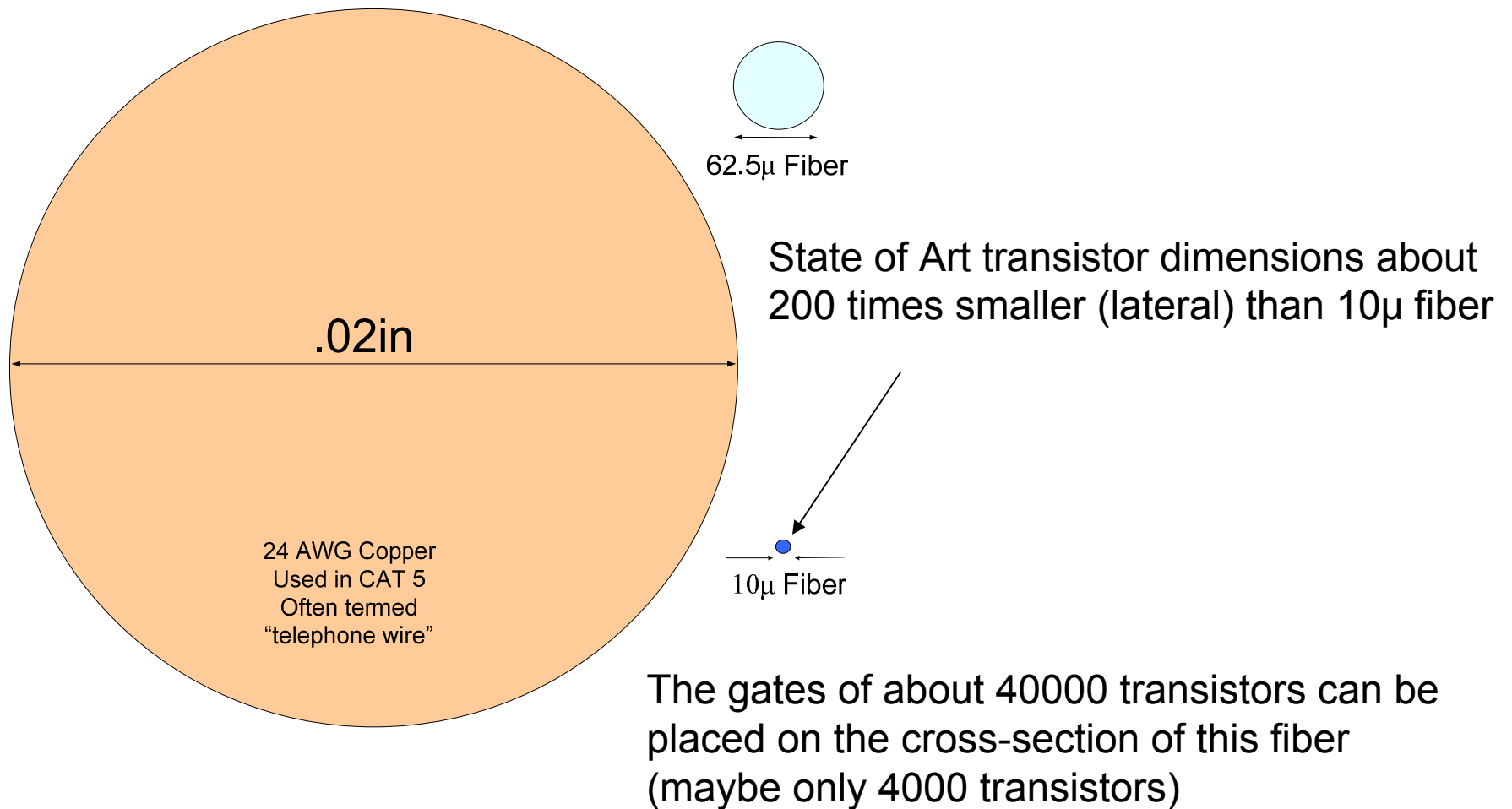
$$I_D = ?$$

$$I_G = 0$$

$$I_B = 0$$

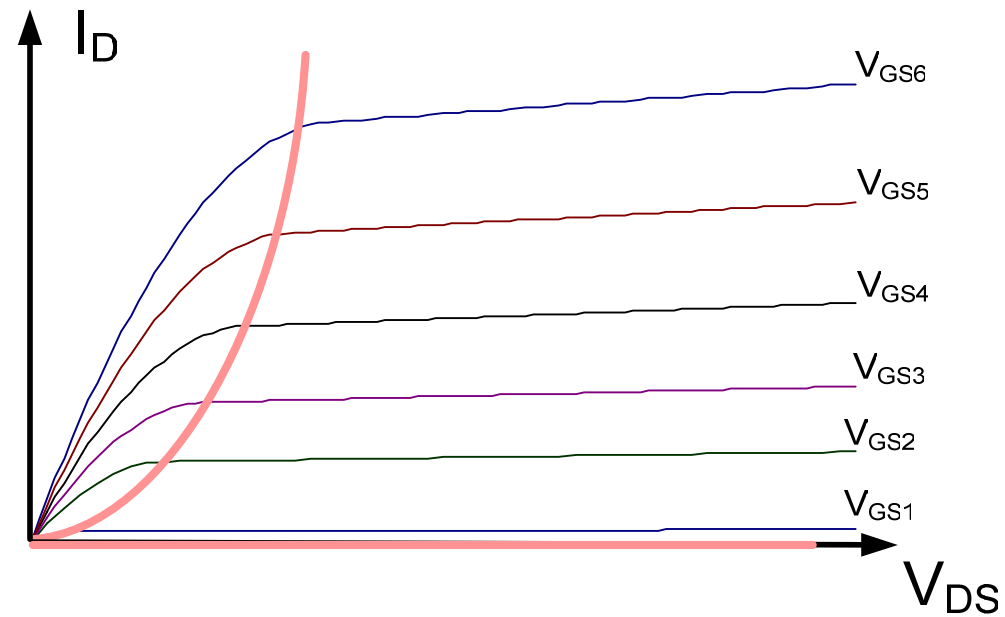
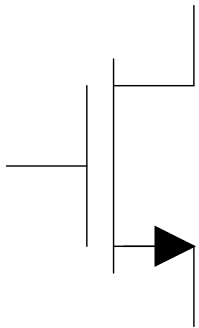
Review from Last Time:

Transistor Size Comparison with 24AWG Copper Cable (Drawn to scale)



Review from Last Time:

MOS Transistors



Standard square-law model

$$I_G = 0$$

$$I_D = \begin{cases} 0 & V_{GS} < V_T & \text{Cutoff} \\ \left(\frac{\mu_n C_{OX} W}{L} \right) \left(V_{GS} - V_T - \frac{V_{DS}}{2} \right) V_{DS} & V_{GS} \geq V_T, V_{DS} \leq V_{GS} - V_T & \text{Triode} \\ \left(\frac{\mu_n C_{OX} W}{2L} \right) (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) & V_{GS} \geq V_T, V_{DS} > V_{GS} - V_T & \text{Saturation} \end{cases}$$

$$\mu_n C_{OX} \approx 10^{-4} \text{ A/V}^2$$

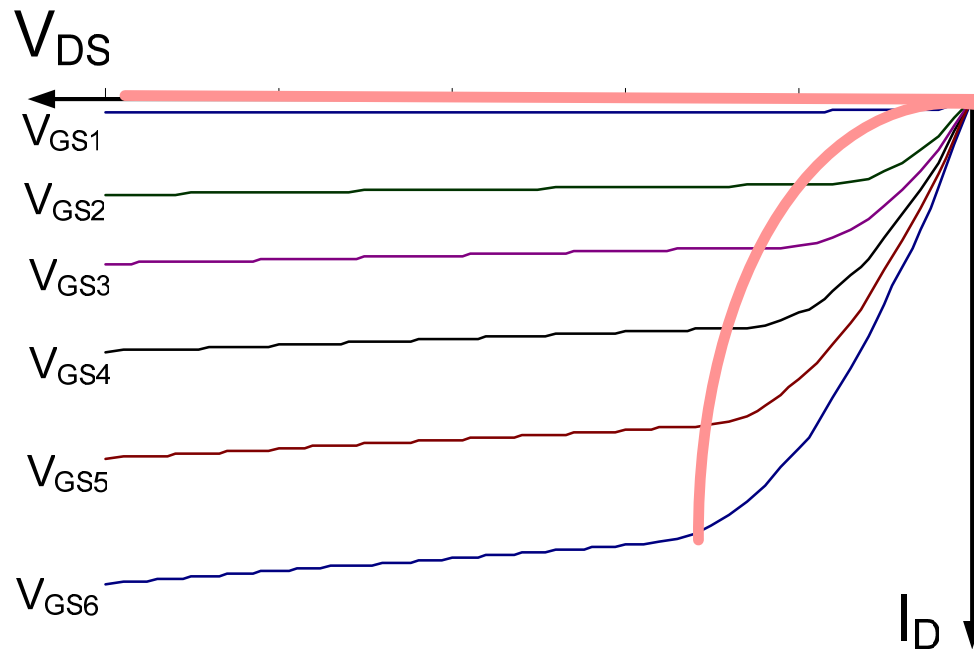
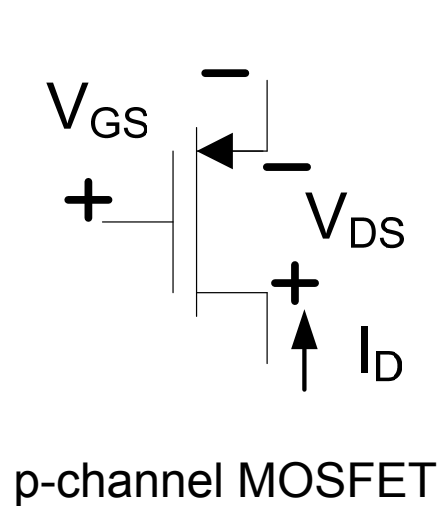
$$\lambda \approx .01 \text{ V}^{-1}$$

$$V_T \approx 0.5 \text{ V to } 3 \text{ V}$$

$$W/L \text{ varies by design}$$

Review from Last Time:

MOS Transistors



Standard square-law model

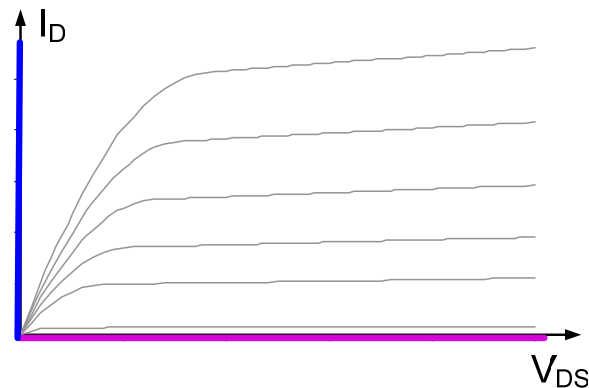
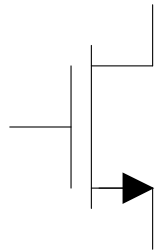
$$I_G = 0$$

$$I_D = \begin{cases} 0 & V_{GS} < V_T \\ -\left(\frac{\mu_p C_{OX} W}{L}\right) \left(V_{GS} - V_T - \frac{V_{DS}}{2}\right) V_{DS} & V_{GS} \leq V_T, V_{DS} \geq V_{GS} - V_T \\ -\left(\frac{\mu_p C_{OX} W}{2L}\right) (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) & V_{GS} \leq V_T, V_{DS} < V_{GS} - V_T \end{cases}$$

	Cutoff	
	Triode	$V_T < 0$
	Saturation	$I_D \leq 0$
		$V_{DS} \leq 0$

Review from Last Time:

MOS Transistor Models simplifications



$$I_G = 0$$

$$I_D = 0$$

$$V_{DS} = 0$$

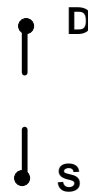
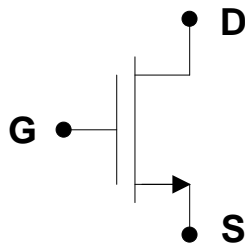
$$V_{GS} < V_T$$

$$V_{GS} \geq V_T$$

Cutoff

Triode

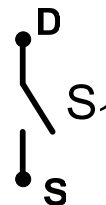
Equivalent Circuit Models



$$V_{GS} < V_T$$



$$V_{GS} > V_T$$

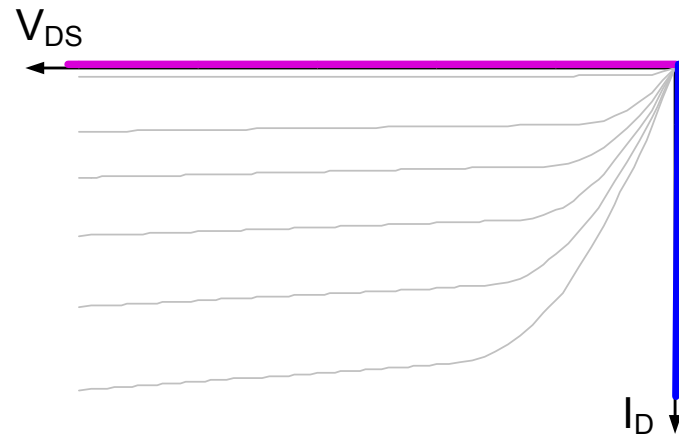
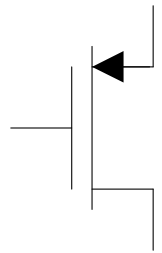


S_1 open for $V_{GS} < V_T$

S_1 closed for $V_{GS} > V_T$

Review from Last Time:

MOS Transistor Models simplifications



$$I_G = 0$$

$$I_D = 0$$

$$V_{DS} = 0$$

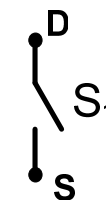
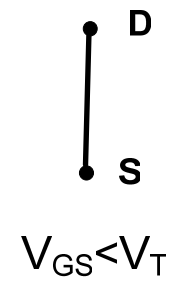
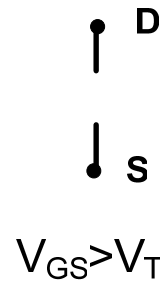
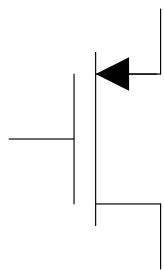
$$V_{GS} > V_T$$

$$V_{GS} \leq V_T$$

Cutoff

Triode

Equivalent Circuit Models

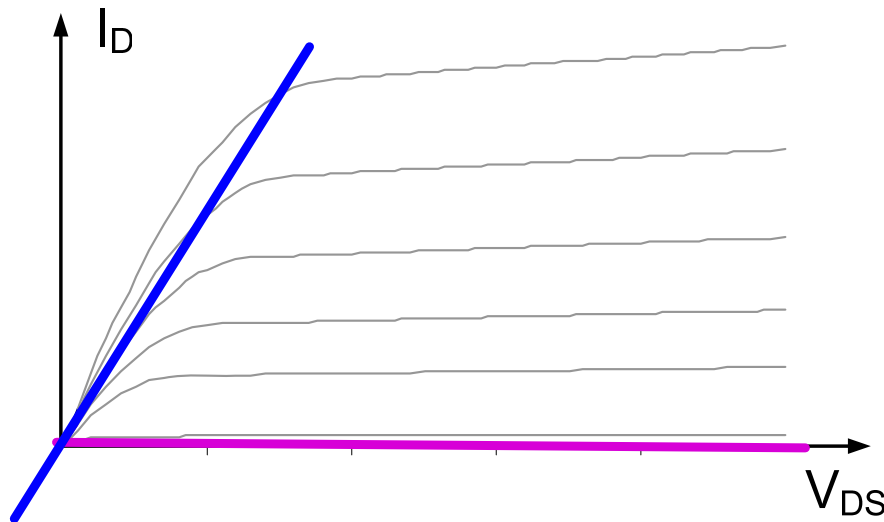
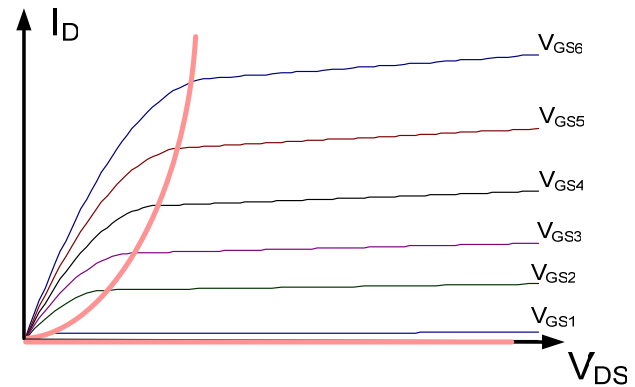
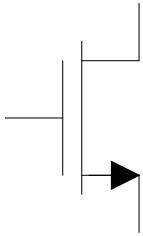


S_1 open for $V_{GS} > V_T$

S_1 closed for $V_{GS} < V_T$

Review from Last Time:

MOS Transistor Models simplifications



$$I_G = 0$$
$$I_D = \begin{cases} 0 & V_{GS} < V_T \\ V_{DS} / R_{FET} & V_{GS} \geq V_T \end{cases}$$

Cutoff

Triode

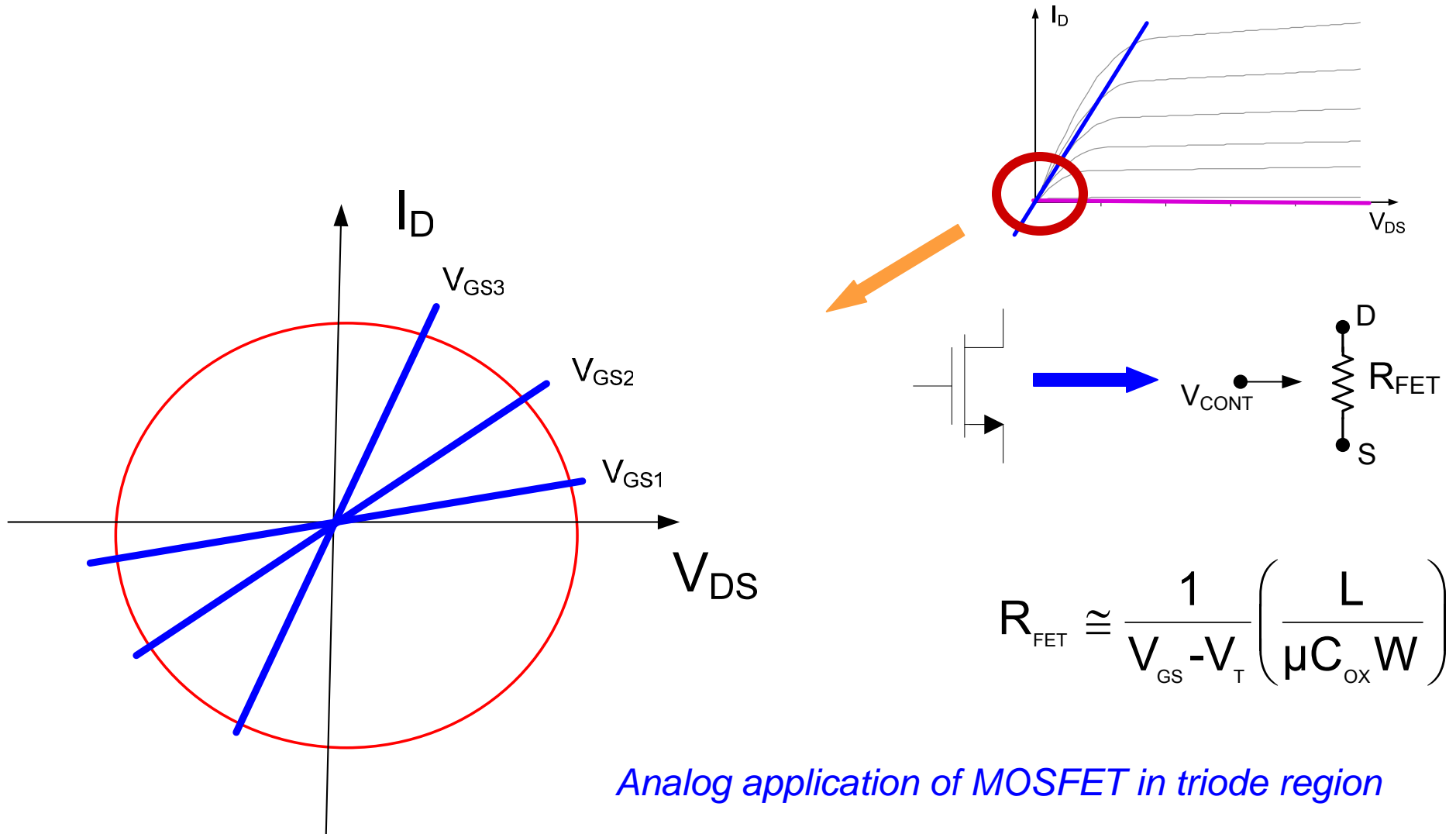
$$R_{FET} \cong \frac{1}{V_{GS} - V_T} \left(\frac{L}{\mu C_{OX} W} \right)$$

Better Switch-level dc model — good enough for predicting basic operation of many digital circuits and can be used to predict speed performance if parasitic capacitances are added

Review from Last Time:

MOS Transistor Models

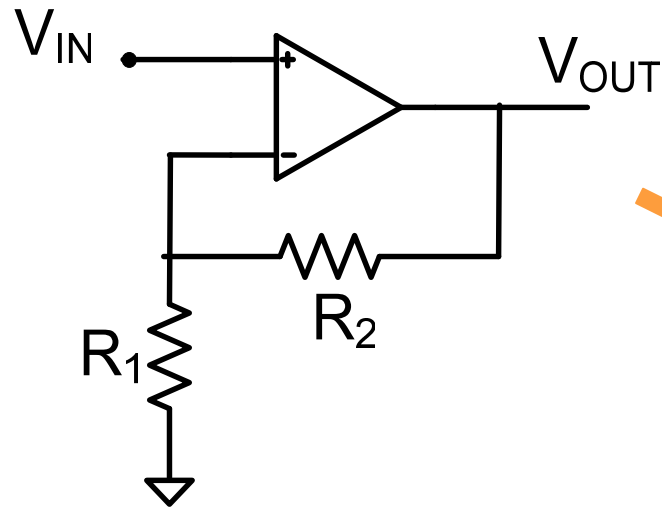
Voltage Variable Resistor (VVR) operation



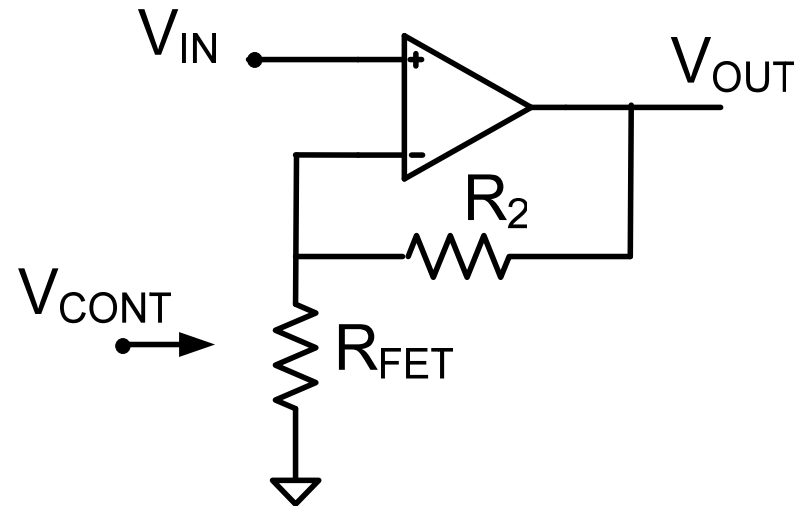
Analog application of MOSFET in triode region

Review from Last Time:

Voltage Variable Resistor



$$A_V = 1 + \frac{R_2}{R_1}$$

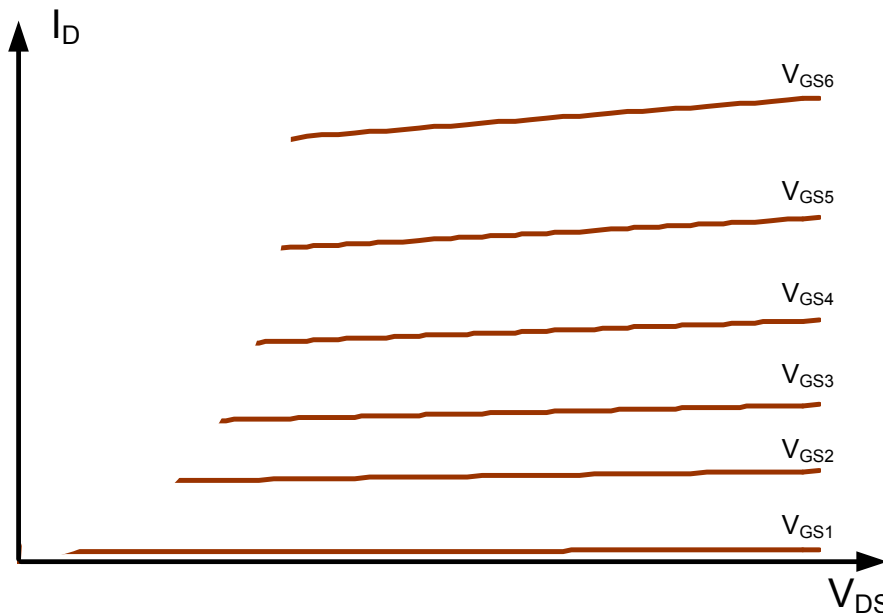
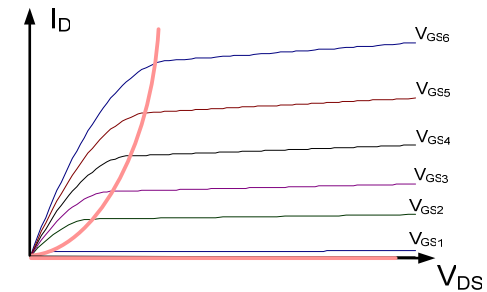
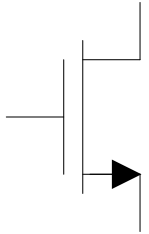


$$A_V = 1 + \frac{R_2}{R_{FET}}$$

$$R_{FET} \cong \frac{1}{V_{GS} - V_T} \left(\frac{L}{\mu C_{OX} W} \right)$$

Applications include Automatic Gain Control (AGC)

MOS Transistor Models simplifications



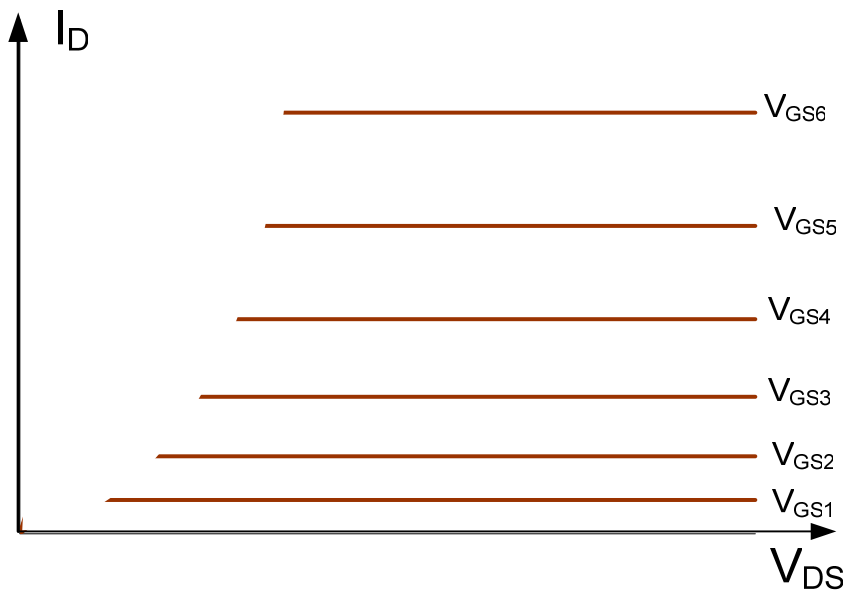
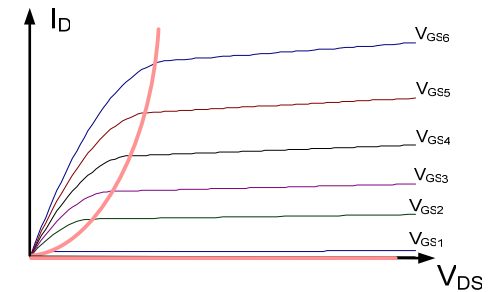
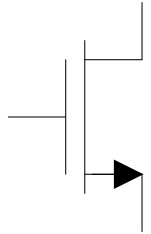
$$I_G = 0$$

$$I_D = \left(\frac{\mu C_{ox} W}{2L} \right) (V_{GS} - V_T)^2 (1 + \lambda V_{DS}) \quad \text{Saturation}$$

Can often assume $\lambda=0$

Saturation Region Model – used for many analog applications

MOS Transistor Models simplifications



$$I_G = 0$$

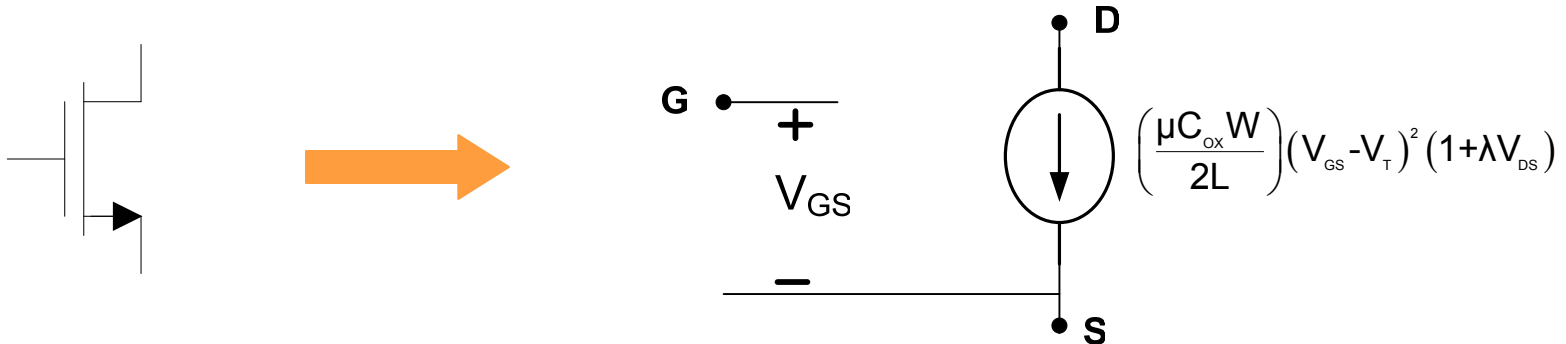
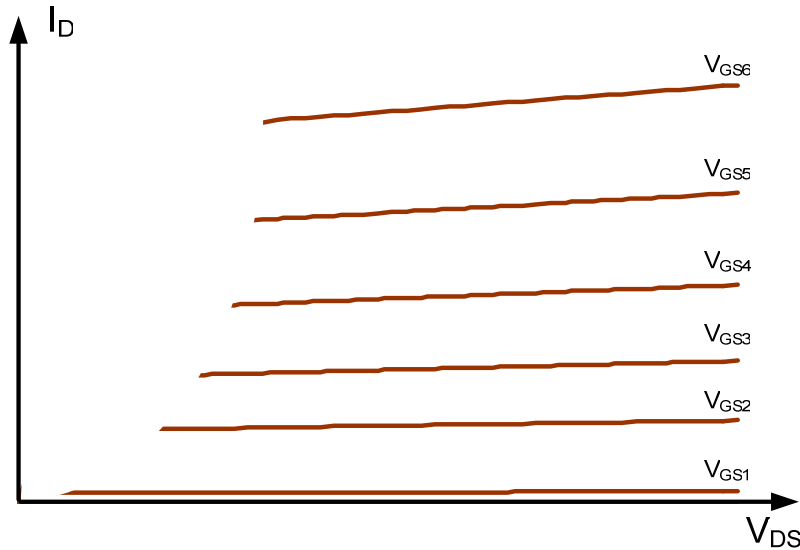
$$I_D = \left(\frac{\mu C_{ox} W}{2L} \right) (V_{GS} - V_T)^2$$

Saturation

With $\lambda=0$

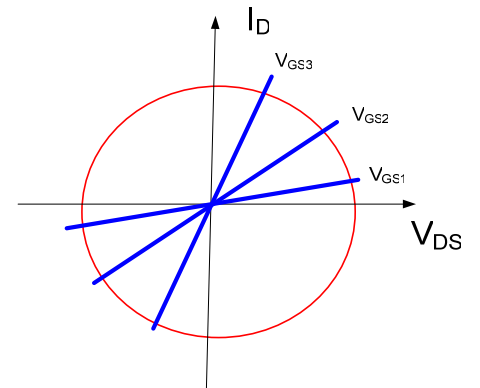
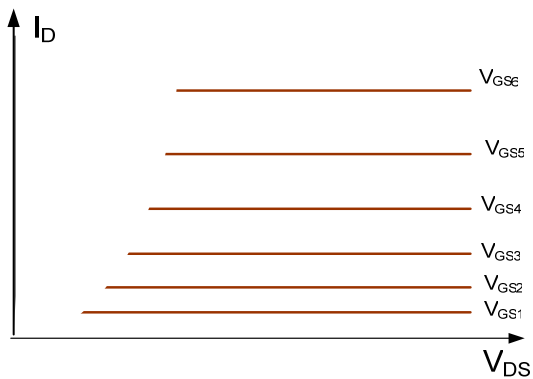
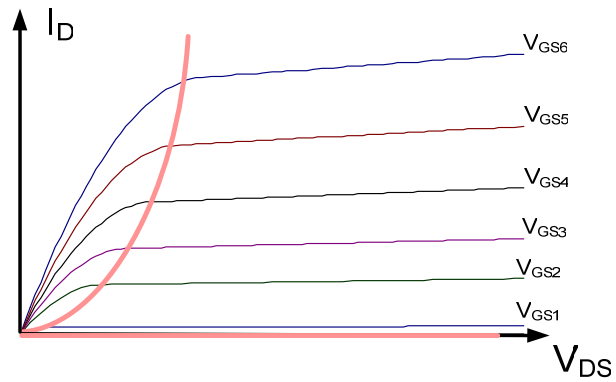
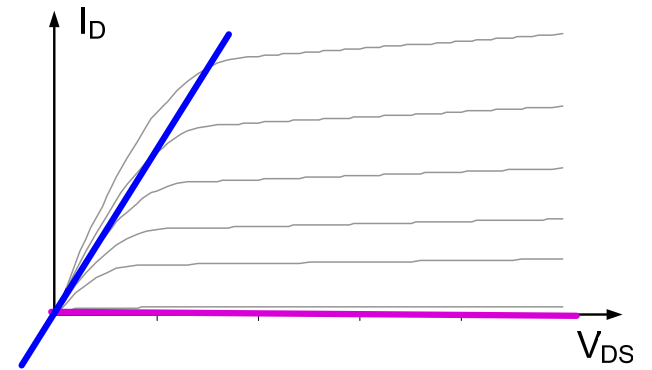
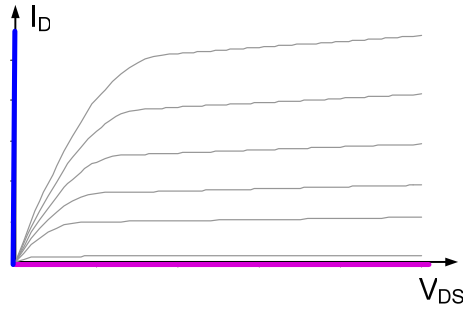
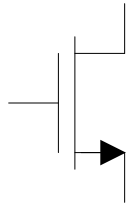
Saturation Region Model — good enough for many analog applications

MOS Transistor Models simplifications

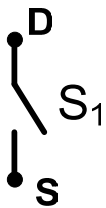
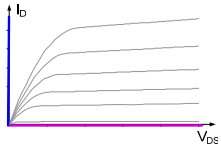
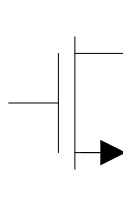


Saturation Region Model — good enough for many analog applications

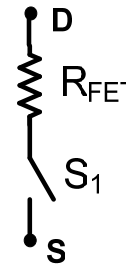
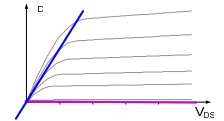
MOS Transistor Models (Summary)



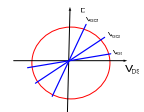
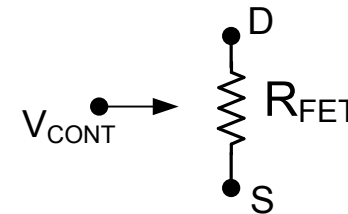
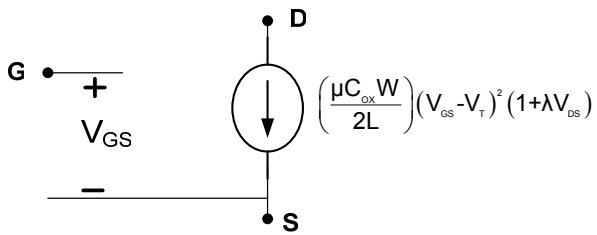
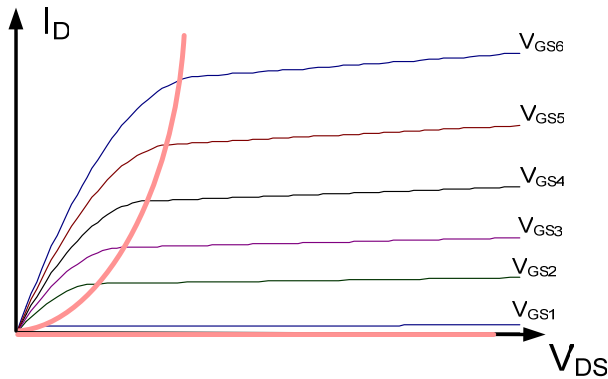
MOS Transistor Models (Summary)



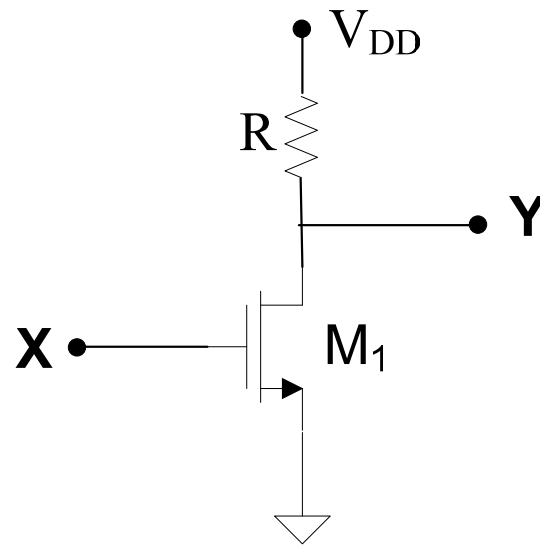
S_1 open for $V_{GS} < V_T$
 S_1 closed for $V_{GS} > V_T$



S_1 closed for $V_{GS} > V_T$
 S_1 open $V_{GS} < V_T$



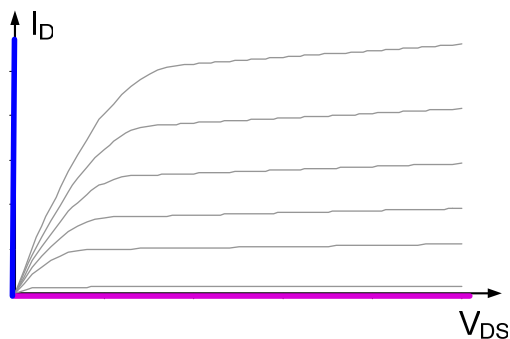
MOS Transistor Applications (Digital Circuits)



Assume "1" ~ $V_H = V_{DD} > V_T$

Assume "0" ~ $V_L = 0V < V_T$

MOSFET Model



$$I_G = 0$$

$$I_D = 0$$

$$V_{DS} = 0$$

$$V_{GS} < V_T$$

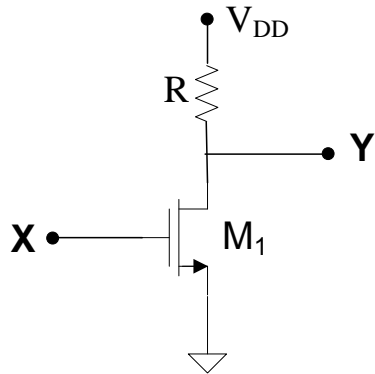
$$V_{GS} \geq V_T$$

Cutoff

Triode

$$\text{Assume } V_T \sim V_{DD}/5$$

MOS Transistor Applications (Digital Circuits)



Assume "1" ~ $V_H = V_{DD} > V_T$

Assume "0" ~ $V_L = 0V < V_T$

$I_D = 0$ $V_{GS} < V_T$ Cutoff

$V_{DS} = 0$ $V_{GS} \geq V_T$ Triode

Assume $V_T \sim V_{DD}/5$

If "1" ~ $X = V_{DD}$, $V_{DS} = 0V$ so $Y = 0V \sim "0"$

If "0" ~ $X = 0V$, $I_D = 0A$ so $Y = V_{DD} - I_D R = V_{DD} \sim "1"$

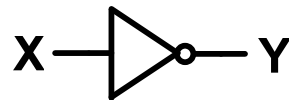
Assume "0" ~ $V_L < V_T$

So this circuit performs as a Boolean inverter

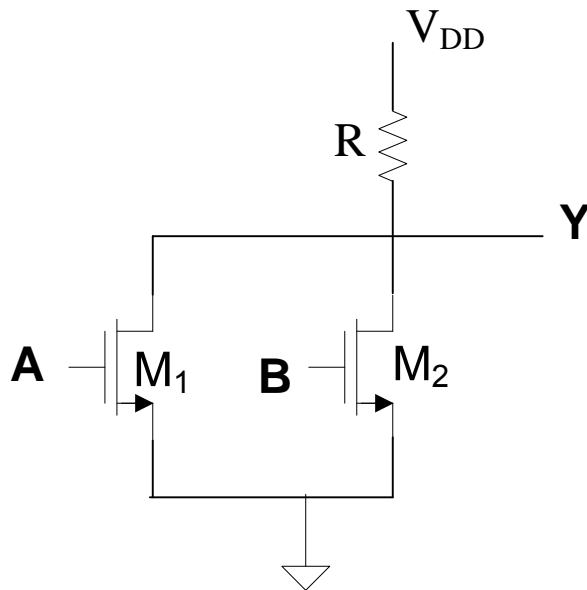
Assume "1" ~ $V_H > V_T$

X	Y
0	1
1	0

Truth Table



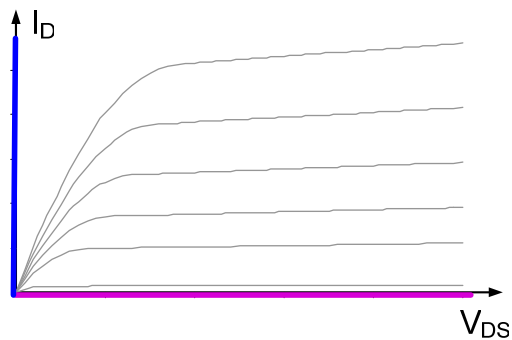
MOS Transistor Applications (Digital Circuits)



Assume "1" ~ $V_H = V_{DD} > V_T$

Assume "0" ~ $V_L = 0V < V_T$

MOSFET Model



$$I_G = 0$$

$$I_D = 0$$

$$V_{DS} = 0$$

$$V_{GS} < V_T$$

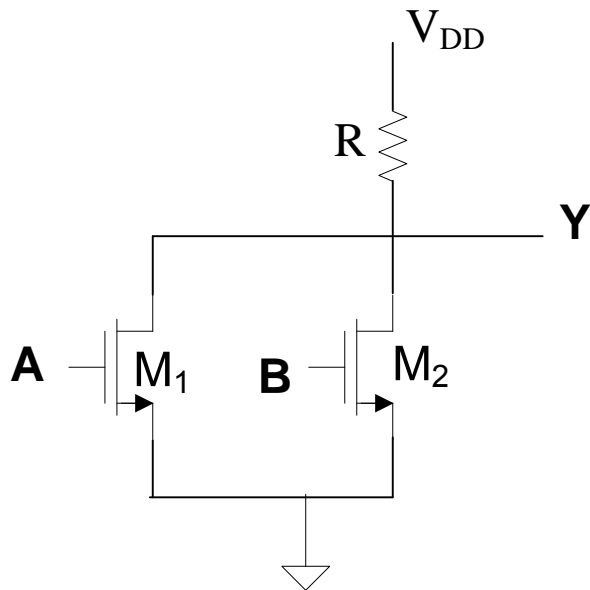
$$V_{GS} \geq V_T$$

Cutoff

Triode

Assume $V_T \sim V_{DD}/5$

MOS Transistor Applications (Digital Circuits)



Assume "1" ~ $V_H = V_{DD} > V_T$

Assume "0" ~ $V_L = 0V < V_T$

$I_D = 0$ $V_{GS} < V_T$ Cutoff

$V_{DS} = 0$ $V_{GS} \geq V_T$ Triode

Assume $V_T \sim V_{DD}/5$

If "1" ~ $A = V_{DD}$, "1" ~ $B = V_{DD}$, $V_{DS1} = V_{DS2} = 0V$ so $Y = 0V \sim "0"$

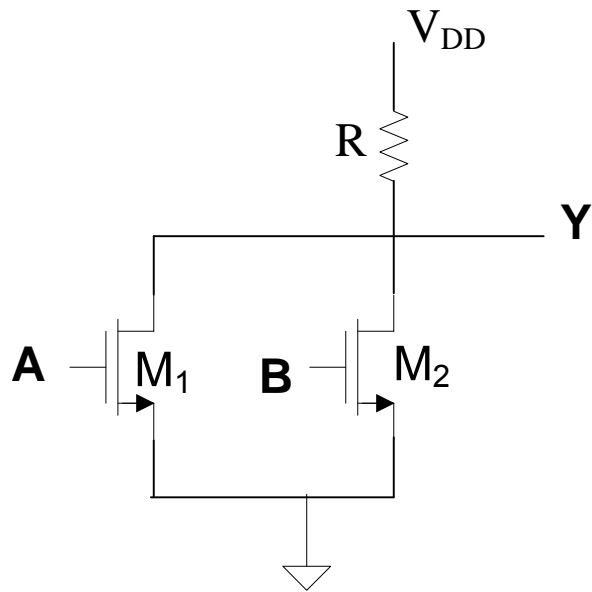
If "1" ~ $A = V_{DD}$, "0" ~ $B = 0V$, $V_{DS1} = 0V$ (and $I_{D2} = 0A$) so $Y = 0V \sim "0"$

If "0" ~ $A = 0V$, "1" ~ $B = V_{DD}$, $V_{DS2} = 0V$ (and $I_{D1} = 0A$) so $Y = 0V \sim "0"$

If "0" ~ $A = 0V$, "0" ~ $B = 0V$, $I_{D2} = 0A$ and $I_{D1} = 0A$ so $I_R = 0A$, thus

$$Y = V_{DD} = I_R R = V_{DD} \sim "1"$$

MOS Transistor Applications (Digital Circuits)



A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

Truth Table

If "1" ~ $A = V_{DD}$, "1" ~ $B = V_{DD}$, $V_{DS1} = V_{DS2} = 0V$ so $Y = 0V \sim "0"$

If "1" ~ $A = V_{DD}$, "0" ~ $B = 0V$, $V_{DS1} = 0V$ (and $I_{D2} = 0A$) so $Y = 0V \sim "0"$

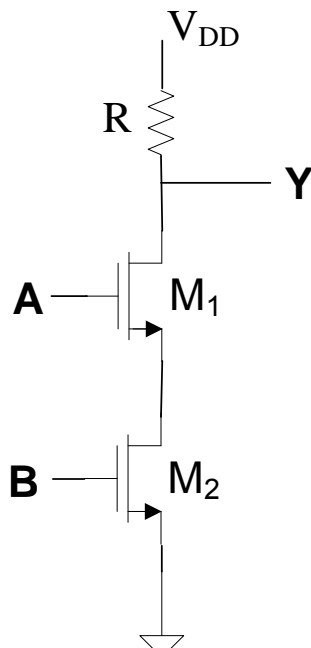
If "0" ~ $A = 0V$, "1" ~ $B = V_{DD}$, $V_{DS2} = 0V$ (and $I_{D1} = 0A$) so $Y = 0V \sim "0"$

If "0" ~ $A = 0V$, "0" ~ $B = 0V$, $I_{D2} = 0A$ and $I_{D1} = 0A$ so $I_R = 0A$, thus

$$Y = V_{DD} = I_R R = V_{DD} \sim "1"$$

2-input NOR Gate

MOS Transistor Applications (Digital Circuits)



Assume "1" ~ $V_H = V_{DD} > V_T$

Assume "0" ~ $V_L = 0V < V_T$

$I_D = 0$ $V_{GS} < V_T$ Cutoff

$V_{DS} = 0$ $V_{GS} \geq V_T$ Triode

Assume $V_T \sim V_{DD}/5$

If "1" ~ $A = V_{DD}$, "1" ~ $B = V_{DD}$, $V_{DS1} = V_{DS2} = 0V$ so $Y = 0V \sim "0"$

If "1" ~ $A = V_{DD}$, "0" ~ $B = 0V$, $V_{DS1} = 0V$ and $I_{D2} = 0A$ so $I_R = 0A$ thus

$$Y = V_{DD} - I_R R = V_{DD} \sim "1"$$

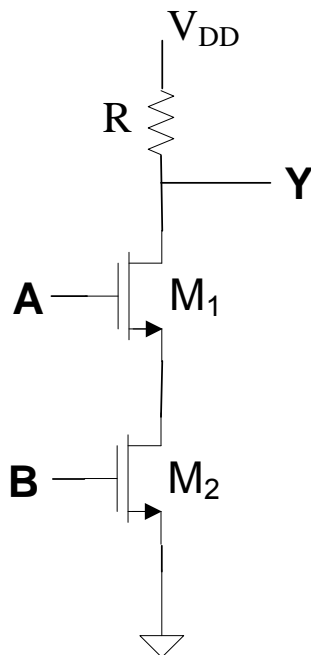
If "0" ~ $A = 0V$, "1" ~ $B = V_{DD}$, $V_{DS2} = 0V$ and $I_{D1} = 0A$ so $I_R = 0A$ thus

$$Y = V_{DD} - I_R R = V_{DD} \sim "1"$$

If "0" ~ $A = 0V$, "0" ~ $B = 0V$, $I_{D1} = 0A$ and $I_{D2} = 0A$ so $I_R = 0A$ thus

$$Y = V_{DD} - I_R R = V_{DD} \sim "1"$$

MOS Transistor Applications (Digital Circuits)



A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

Truth Table

If "1" ~ $A = V_{DD}$, "1" ~ $B = V_{DD}$, $V_{DS1} = V_{DS2} = 0V$ so $Y = 0V \sim "0"$

If "1" ~ $A = V_{DD}$, "0" ~ $B = 0V$, $V_{DS1} = 0V$ and $I_{D2} = 0A$ so $I_R = 0A$ thus

$$Y = V_{DD} - I_R R = V_{DD} \sim "1"$$

If "0" ~ $A = 0V$, "1" ~ $B = V_{DD}$, $V_{DS2} = 0V$ and $I_{D1} = 0A$ so $I_R = 0A$ thus

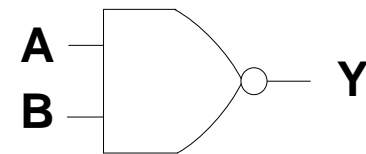
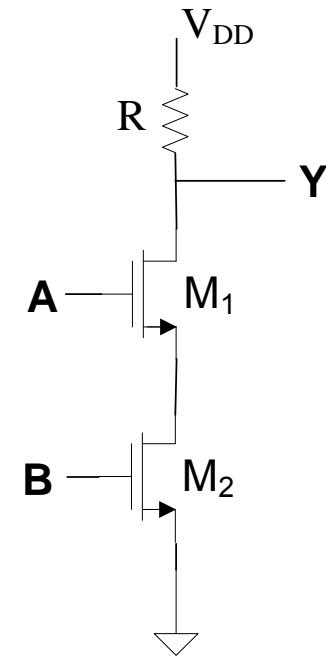
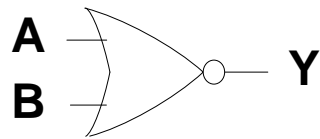
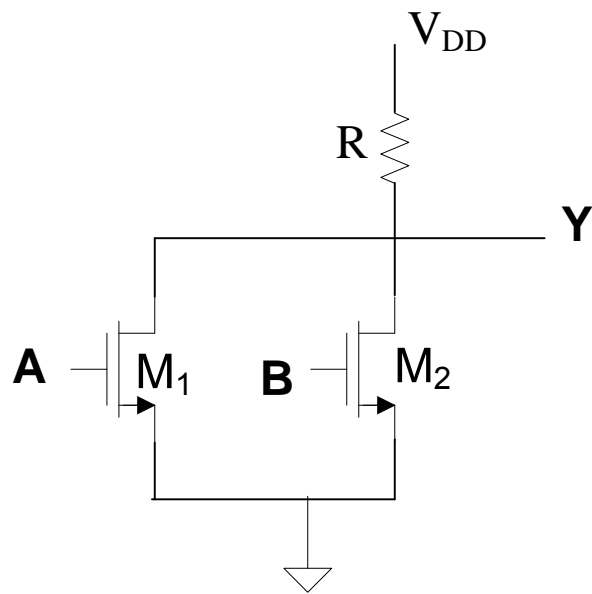
$$Y = V_{DD} - I_R R = V_{DD} \sim "1"$$

If "0" ~ $A = 0V$, "0" ~ $B = 0V$, $I_{D1} = 0A$ and $I_{D2} = 0A$ so $I_R = 0A$ thus

$$Y = V_{DD} - I_R R = V_{DD} \sim "1"$$

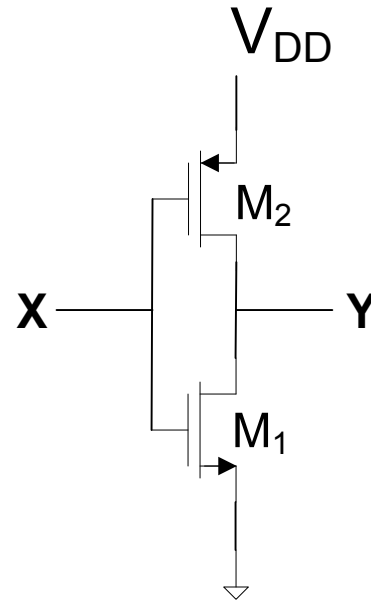
2-input NAND Gate

MOS Transistor Applications (Digital Circuits)



- Can be extended to arbitrary number of inputs
- But the resistor is not practically available in most processes and static power dissipation is too high

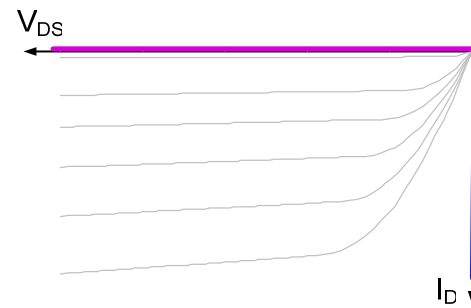
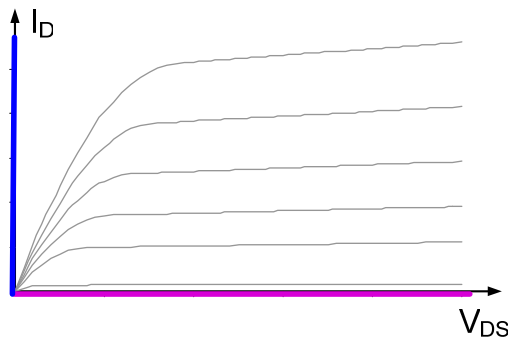
MOS Transistor Applications (Digital Circuits)



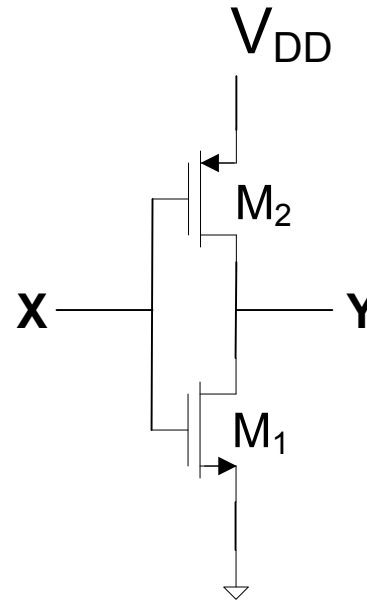
Assume "1" ~ $V_H = V_{DD}$

Assume "0" ~ $V_L = 0V$

MOSFET Models



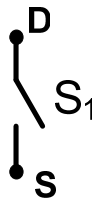
MOS Transistor Applications (Digital Circuits)



Assume "1" ~ $V_H = V_{DD}$

Assume "0" ~ $V_L = 0V$

MOSFET Models

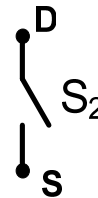


S_1 open for $V_{GS1} < V_{T1}$

S_1 closed for $V_{GS1} > V_{T1}$

Assume $V_{T1} \sim V_{DD}/5$

n-channel device



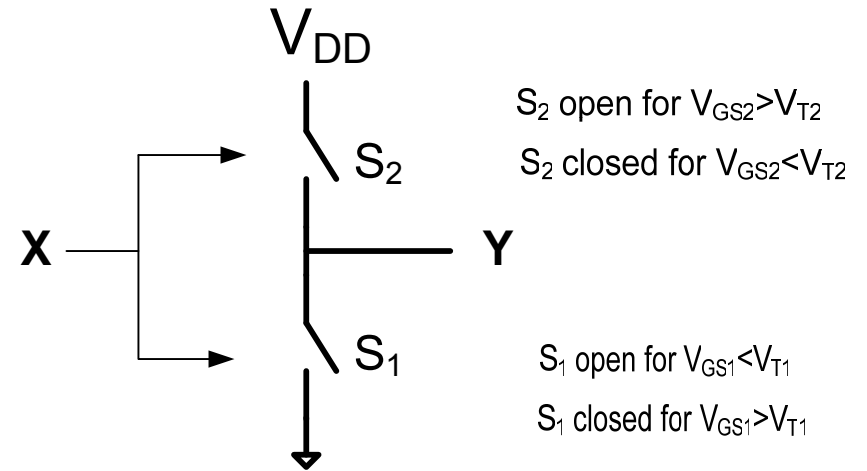
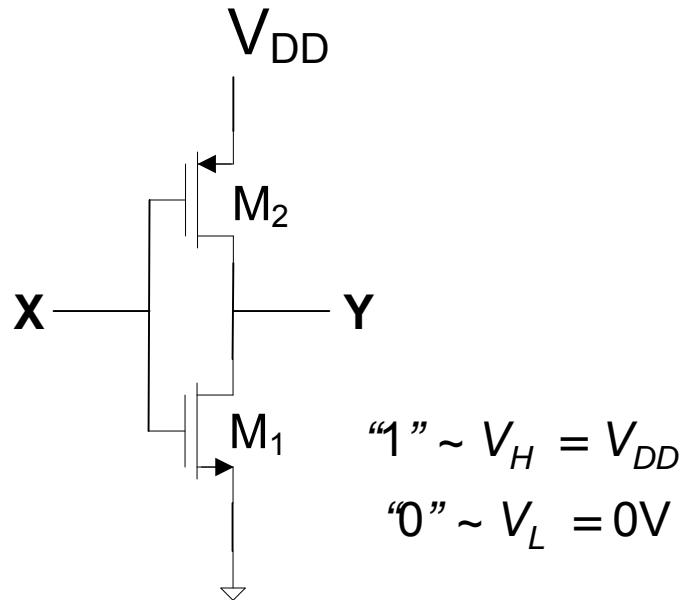
S_2 open for $V_{GS2} > V_{T2}$

S_2 closed for $V_{GS2} < V_{T2}$

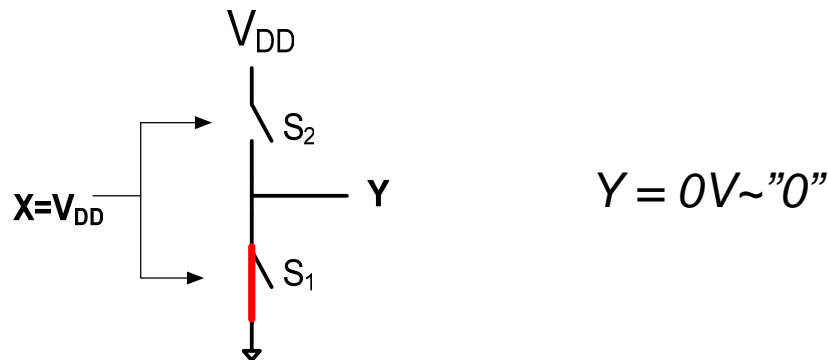
Assume $V_{T2} \sim -V_{DD}/5$

p-channel device

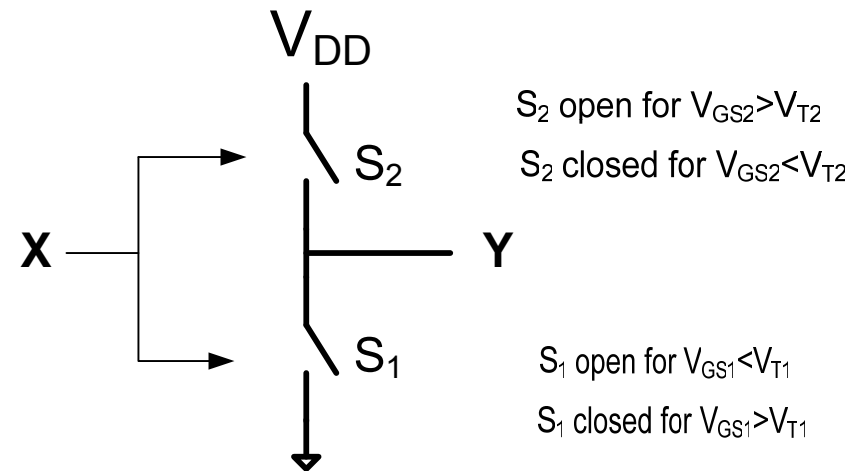
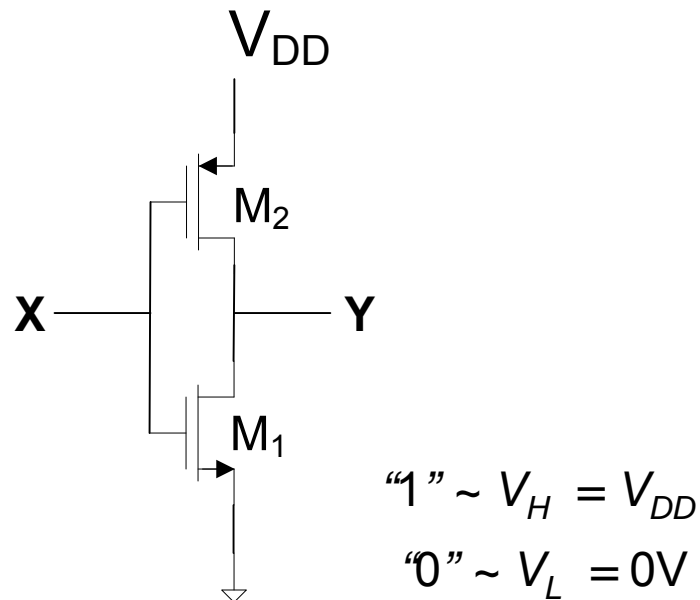
MOS Transistor Applications (Digital Circuits)



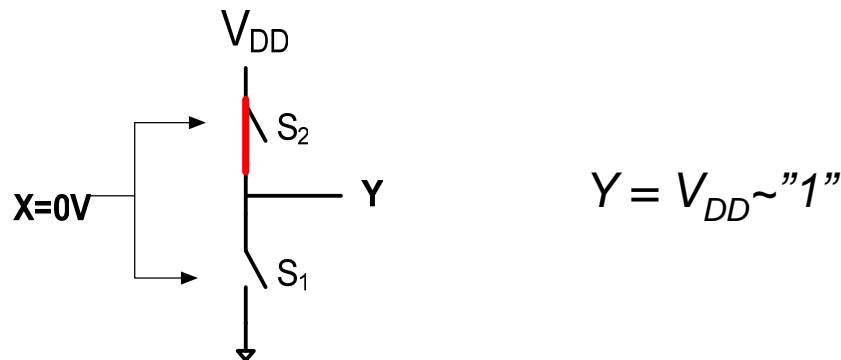
If $X = V_{DD}$, then $V_{GS1} = V_{DD} > V_{T1}$, $V_{GS2} = 0 < V_{T2}$ \longrightarrow S_1 closed, S_2 open



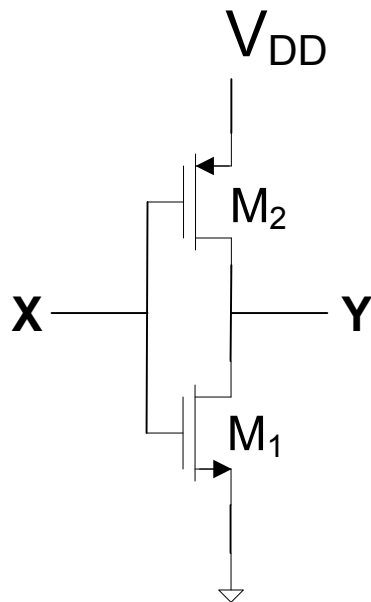
MOS Transistor Applications (Digital Circuits)



If $X=0V$, then $V_{GS1}=0V < V_{T1}$, $V_{GS2}=-V_{DD} < V_{T2}$ \longrightarrow S_2 closed, S_1 open



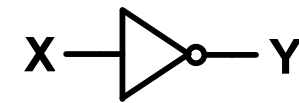
MOS Transistor Applications (Digital Circuits)



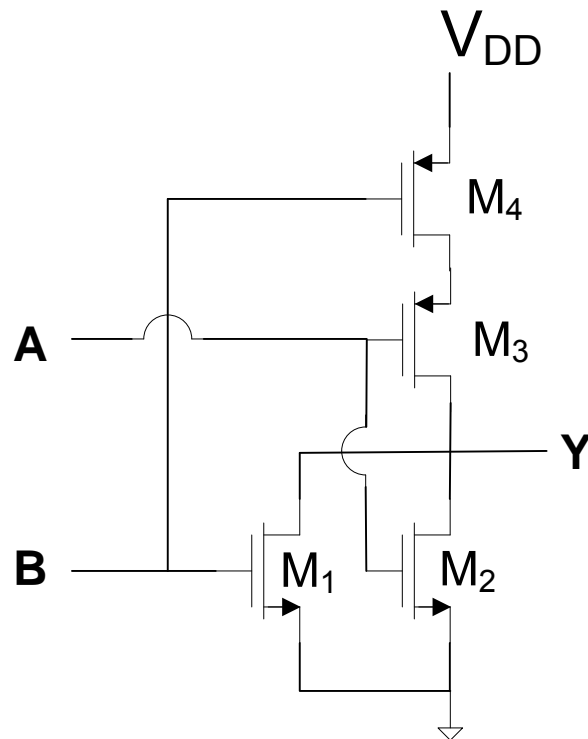
Performs as a digital inverter

X	Y
0	1
1	0

Truth Table



MOS Transistor Applications (Digital Circuits)

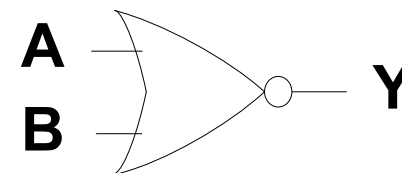


A	B	Y
0	0	1
0	1	0
1	0	0
1	1	0

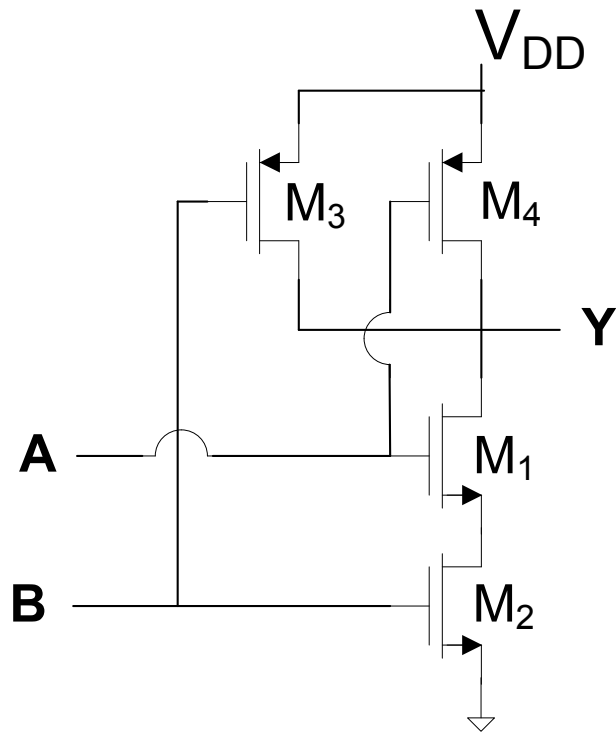
Truth Table

Performs as a 2-input NOR Gate

Can be easily extended to an n-input NOR Gate



MOS Transistor Applications (Digital Circuits)

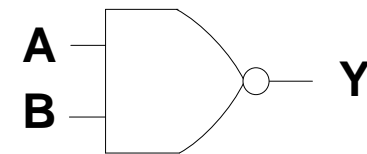


A	B	Y
0	0	1
0	1	1
1	0	1
1	1	0

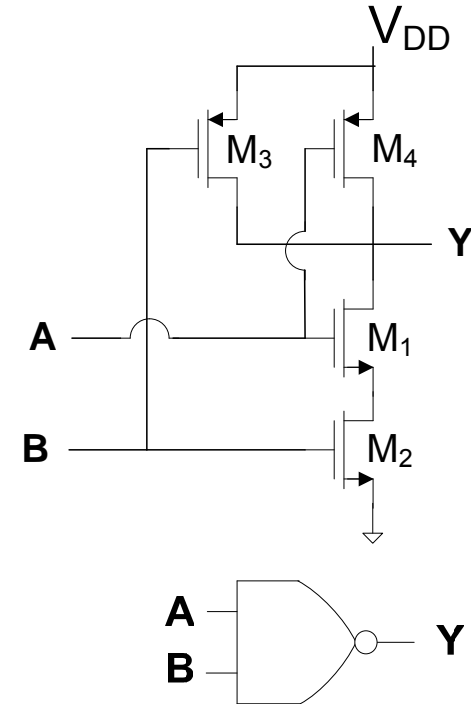
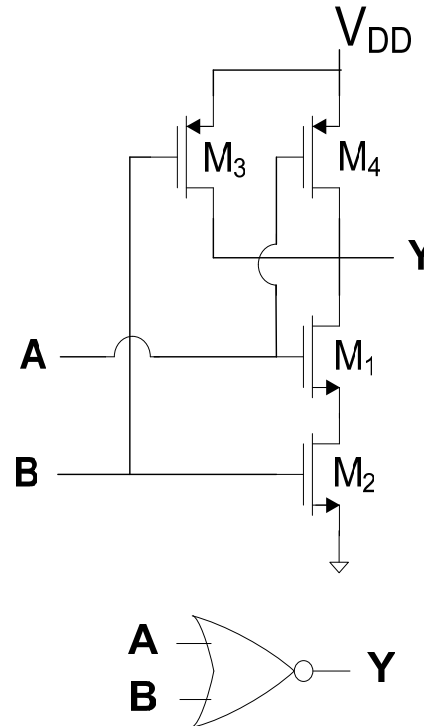
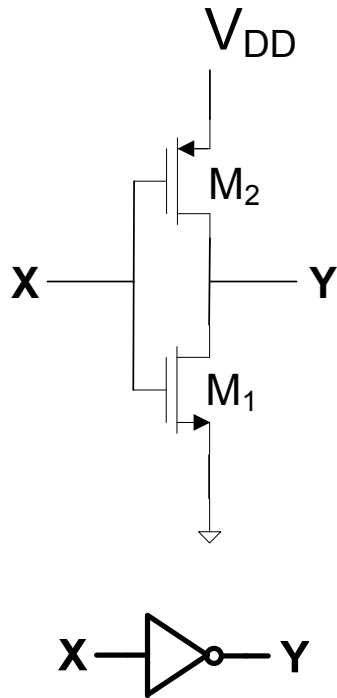
Truth Table

Performs as a 2-input NAND Gate

Can be easily extended to an n-input NAND Gate

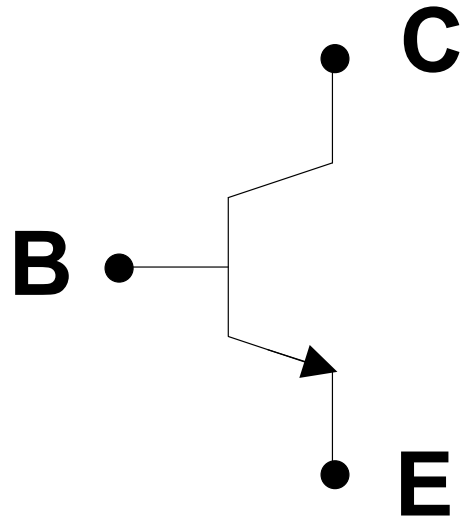


MOS Transistor Applications (Digital Circuits)



- Termed CMOS Logic
- Widely used in industry today (millions of transistors in many ICs using this logic)
- Almost never used as discrete devices

Bipolar Transistor

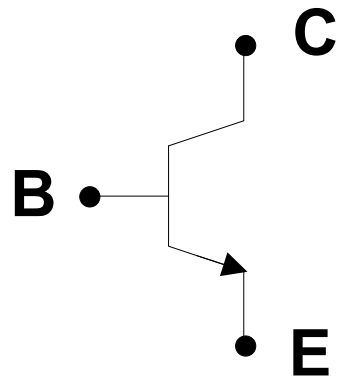


B: Base

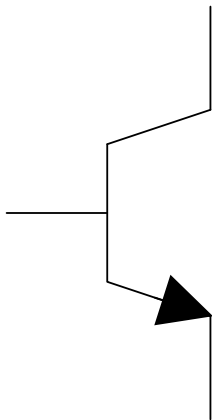
C: Collector

E: Emitter

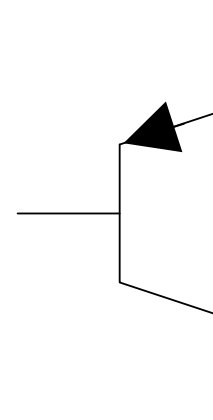
Bipolar Transistor



nnp

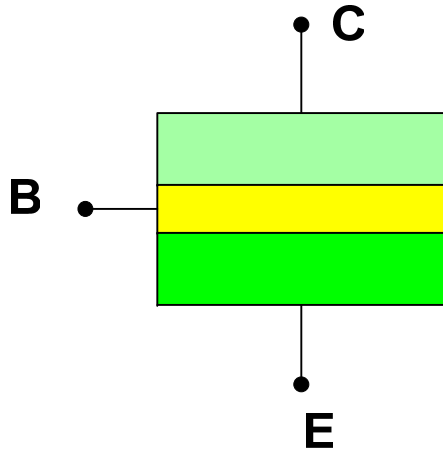
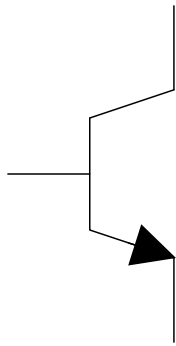


pnnp

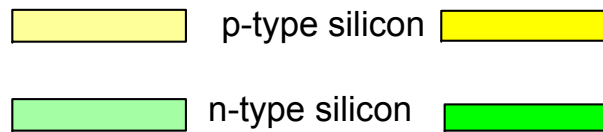
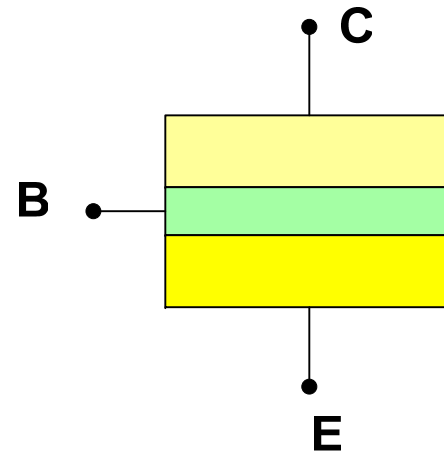
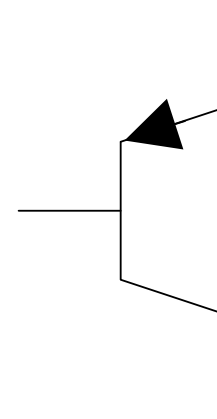


Bipolar Transistor

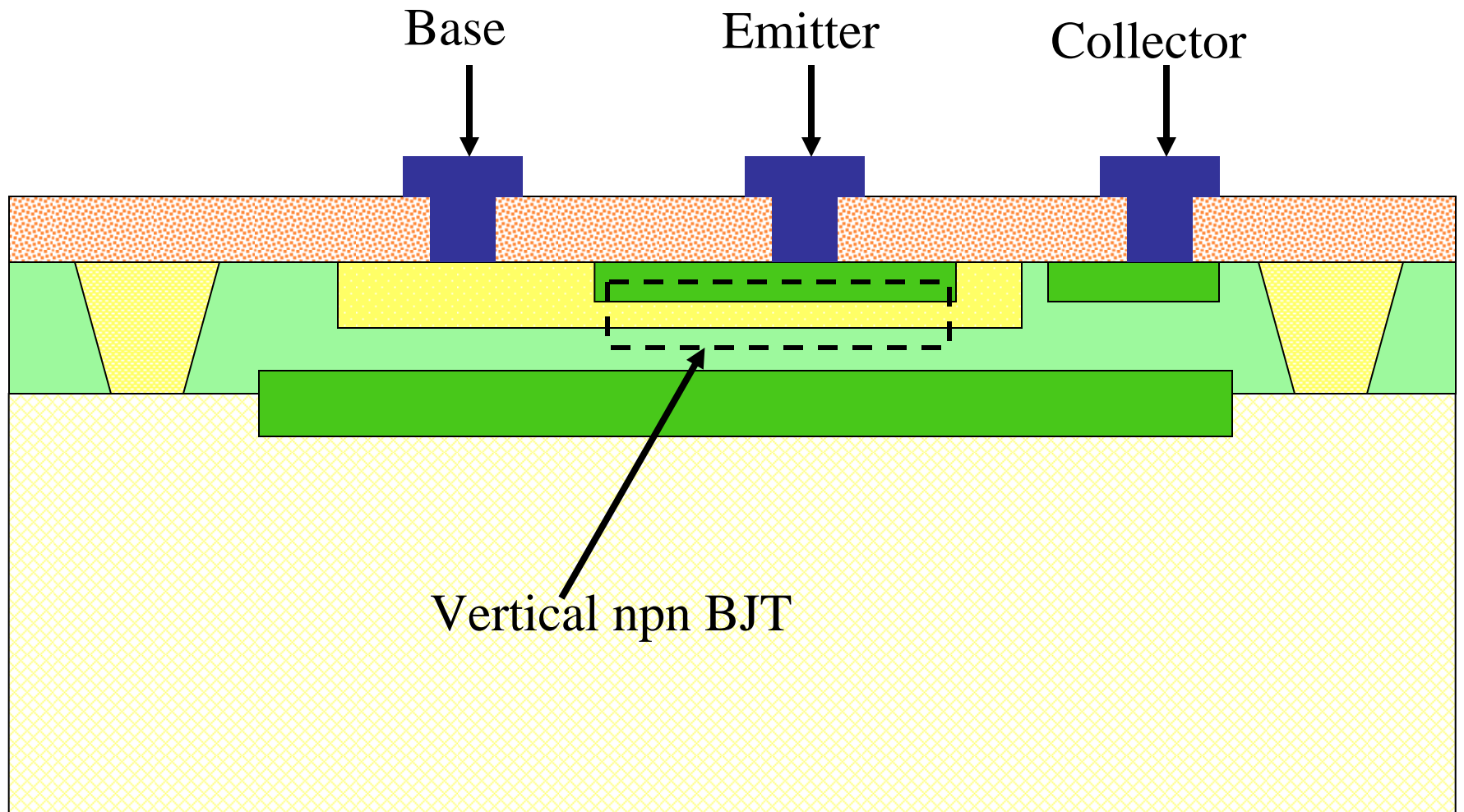
npn



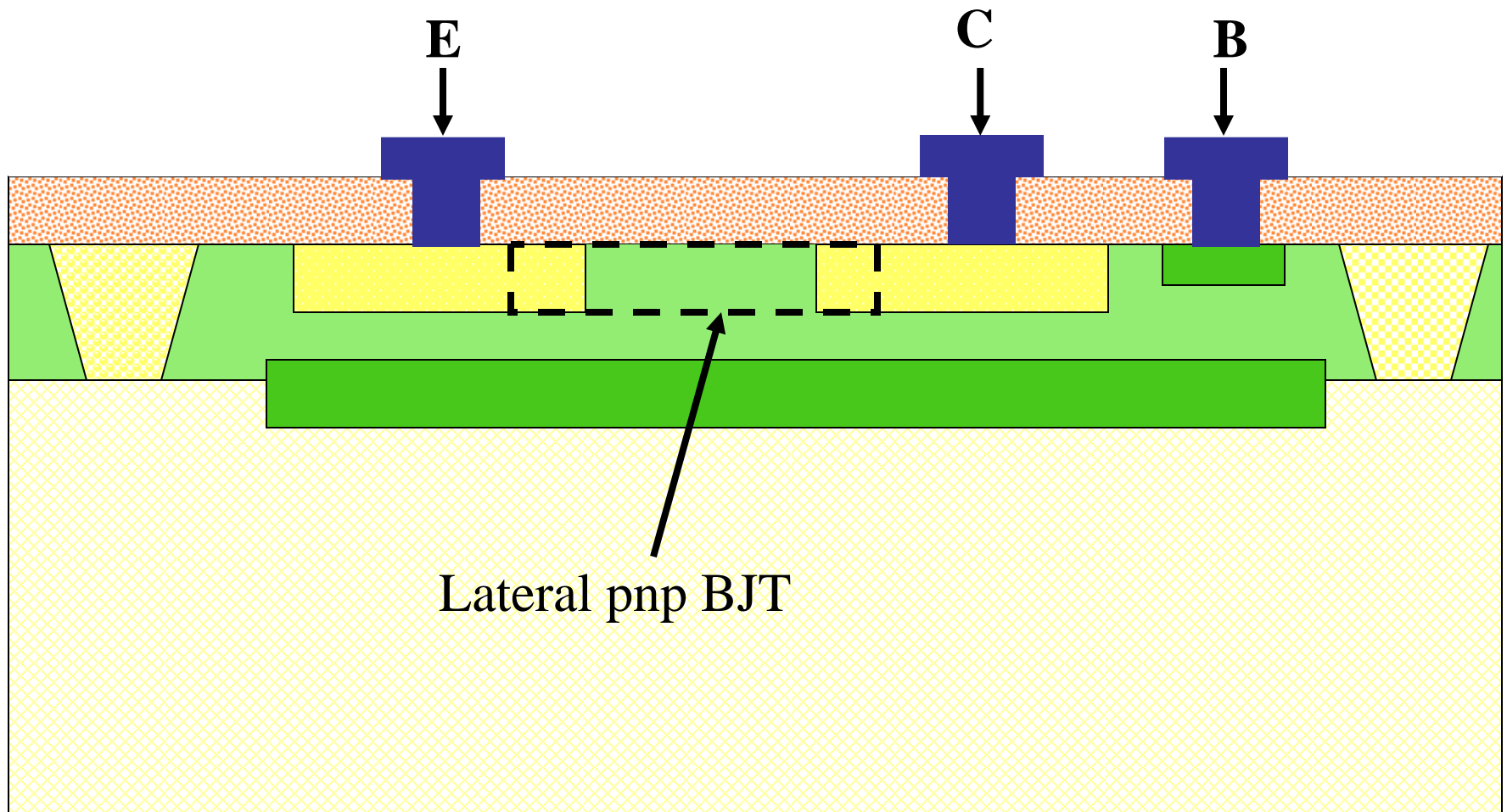
pnp



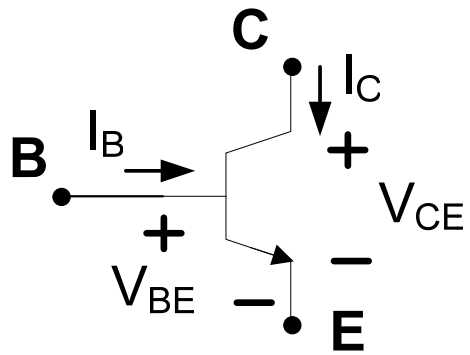
Vertical npn BJT



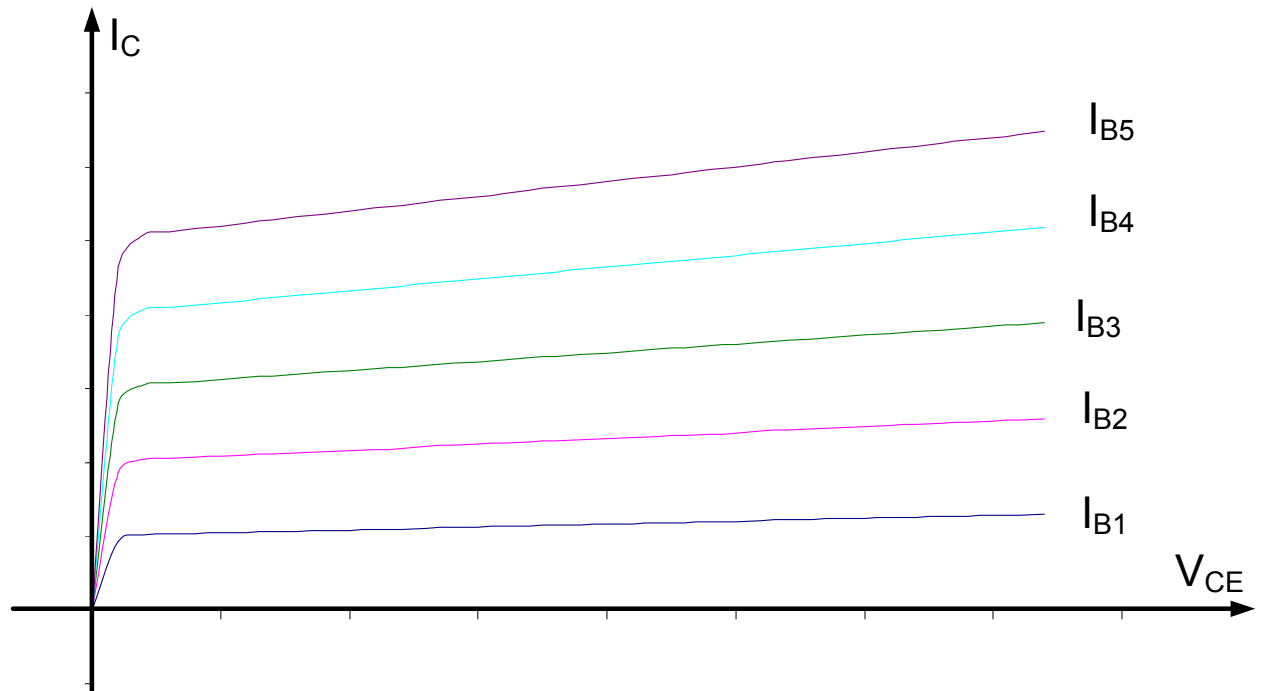
Lateral pnp BJT



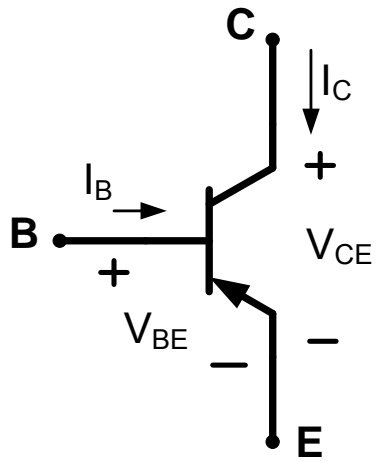
Bipolar Transistor



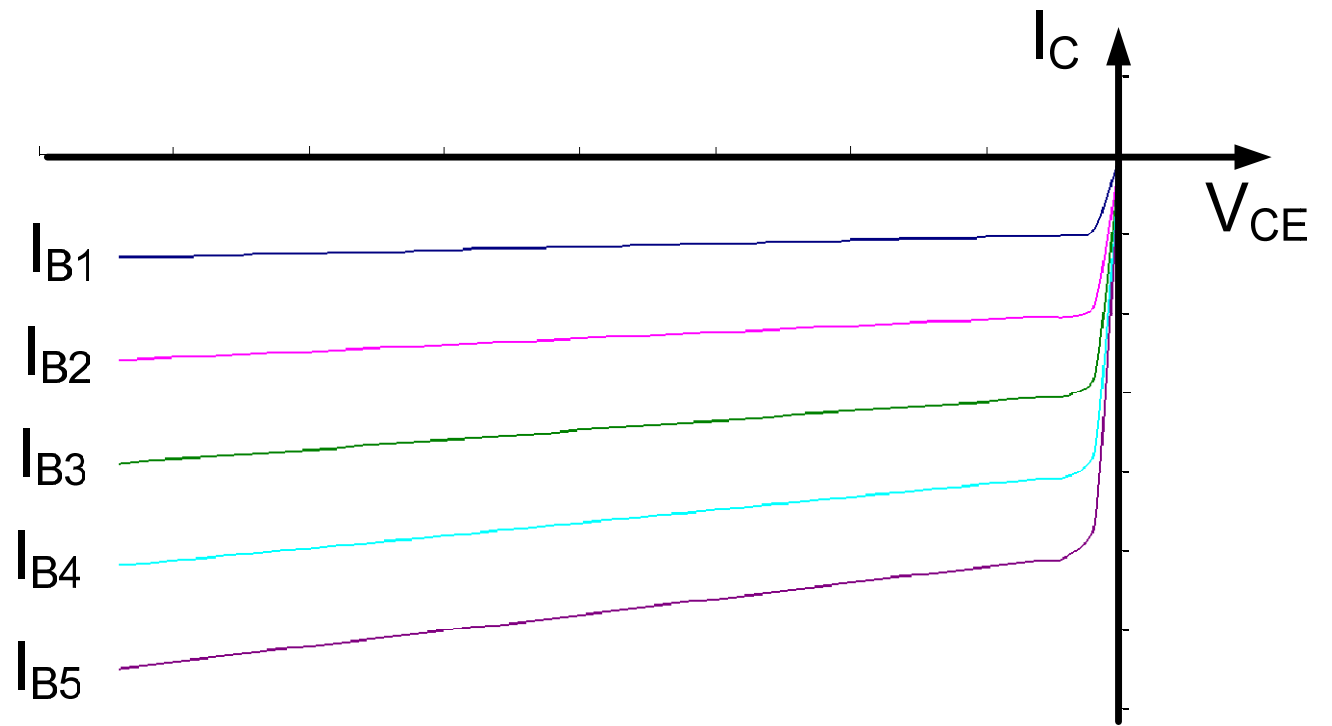
npn



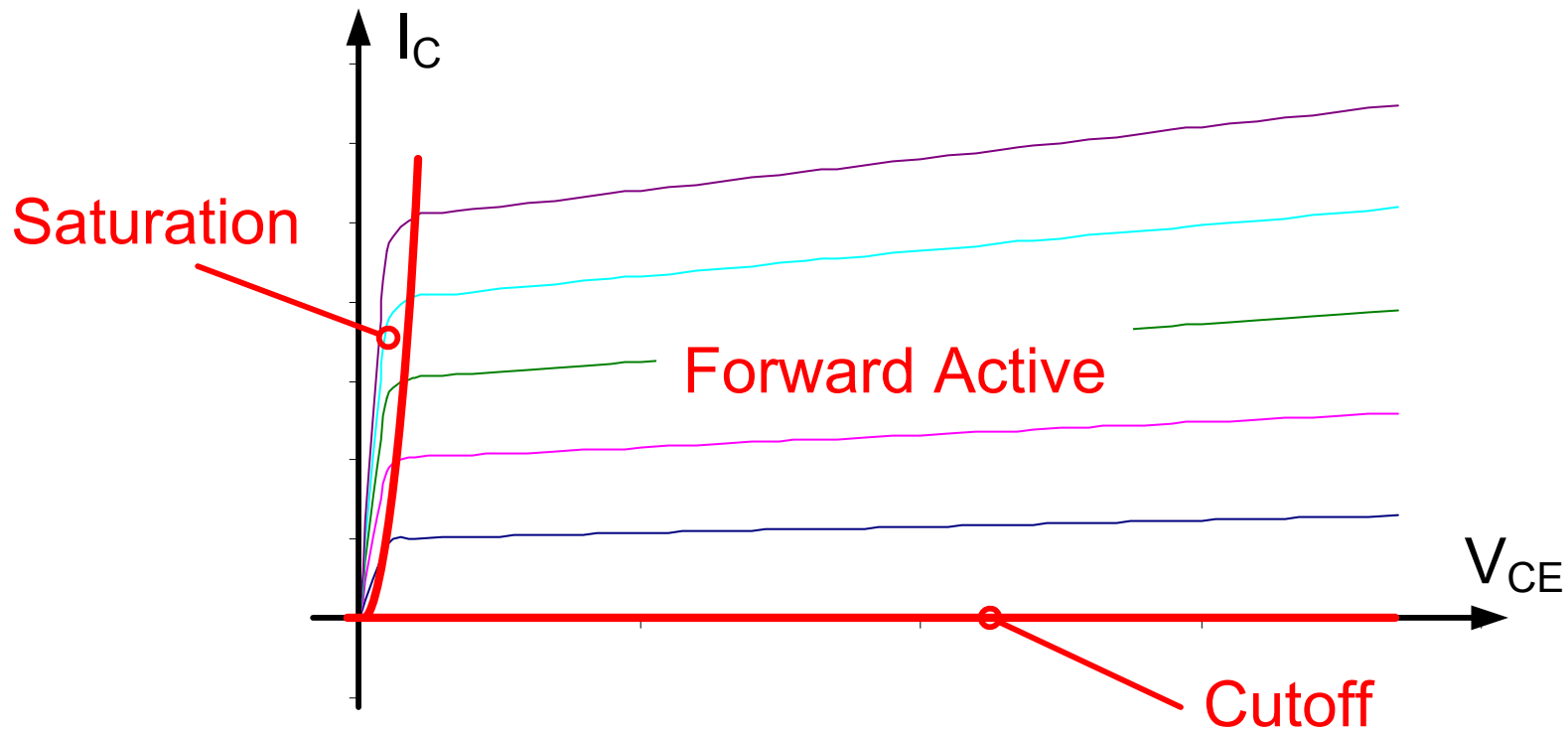
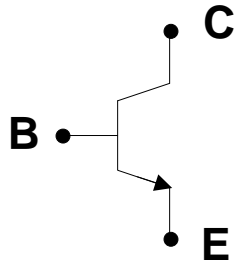
Bipolar Transistor



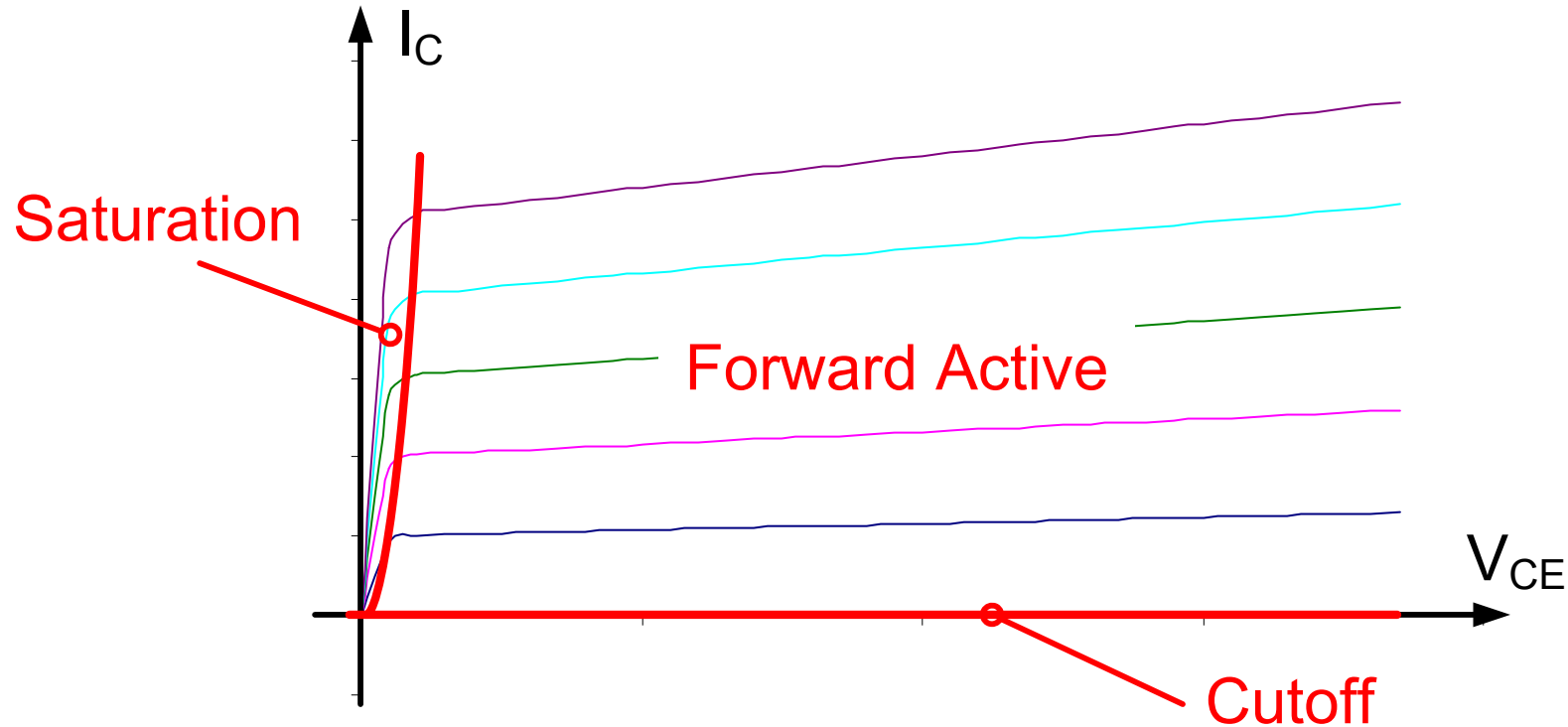
pnp



Bipolar Transistor



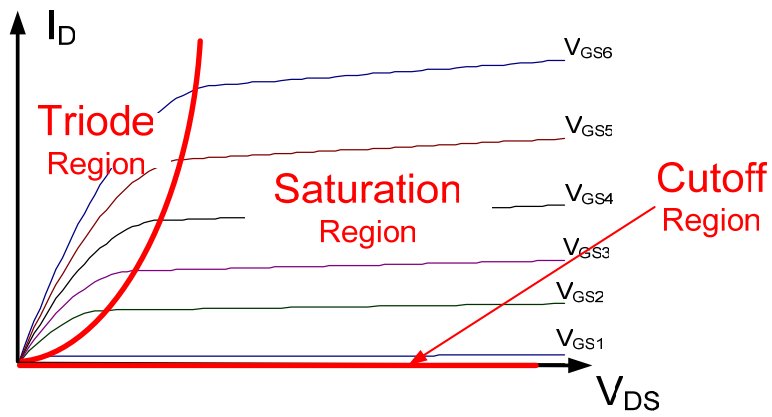
Bipolar Transistor



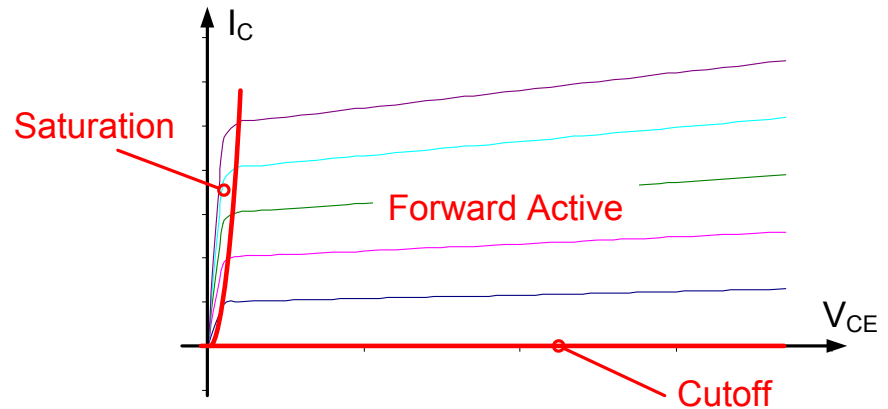
Most analog or linear applications based upon Forward Active region

Most digital applications involve Saturation and Cutoff regions and switching between these regions as the Boolean value changes states

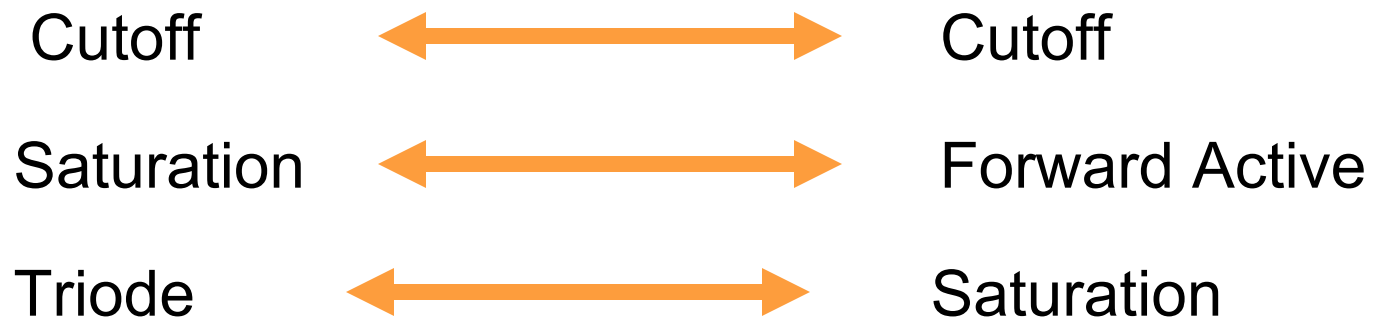
Bipolar and MOS Region Comparisons



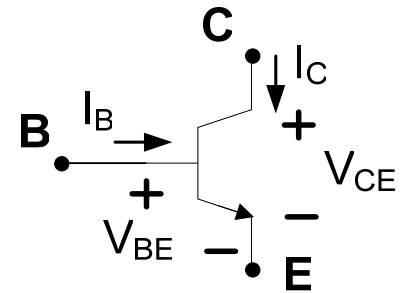
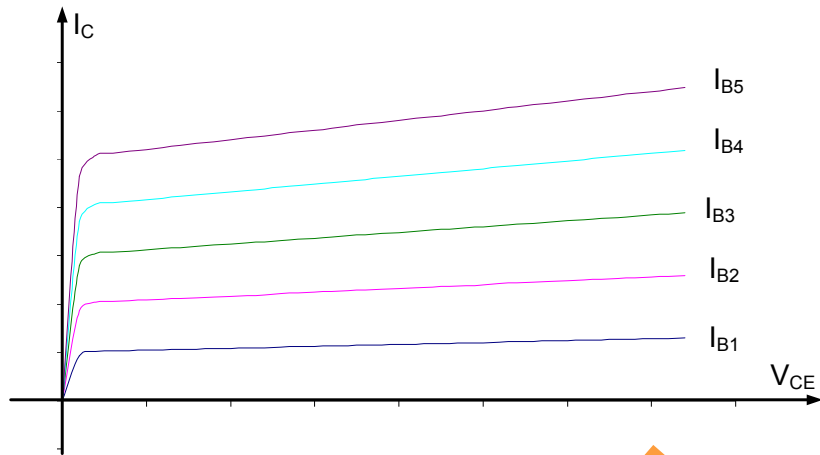
MOSFET



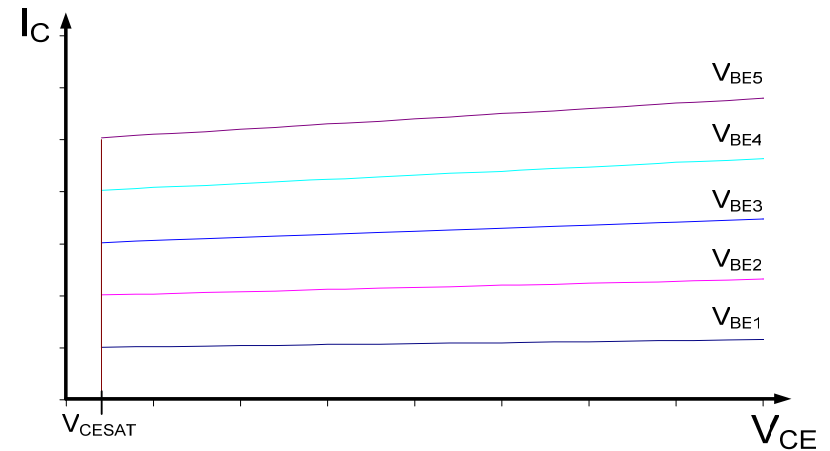
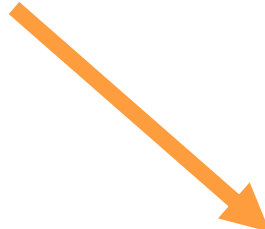
BJT



Bipolar Transistor



npn



Bipolar Transistor

Multi-Region Model

$$I_C = \beta I_B \left(1 + \frac{V_{CE}}{V_{AF}} \right)$$

$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$V_t = \frac{kT}{q}$$

$$V_{BE} > 0.4V$$

$$V_{BC} < 0$$

Forward Active

$$V_{BE} = 0.7V$$
$$V_{CE} = 0.2V$$

$$I_C < \beta I_B$$

Saturation

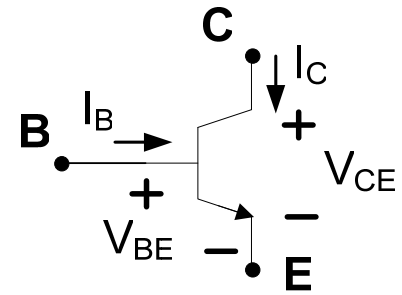
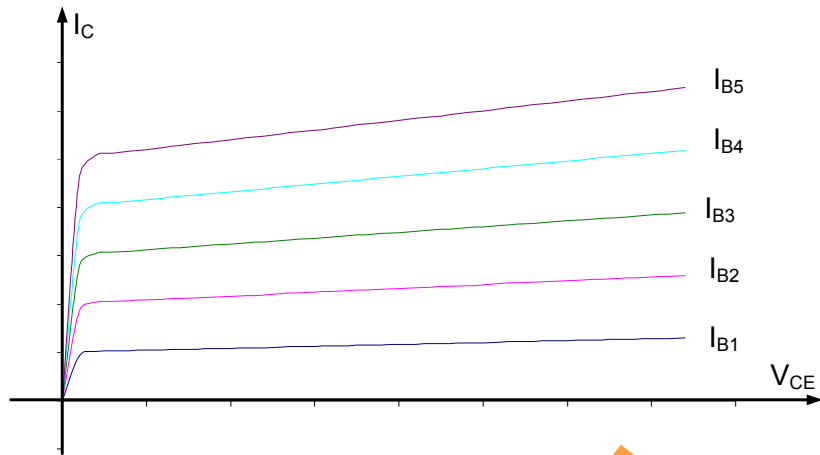
$$I_C = I_B = 0$$

$$V_{BE} < 0$$

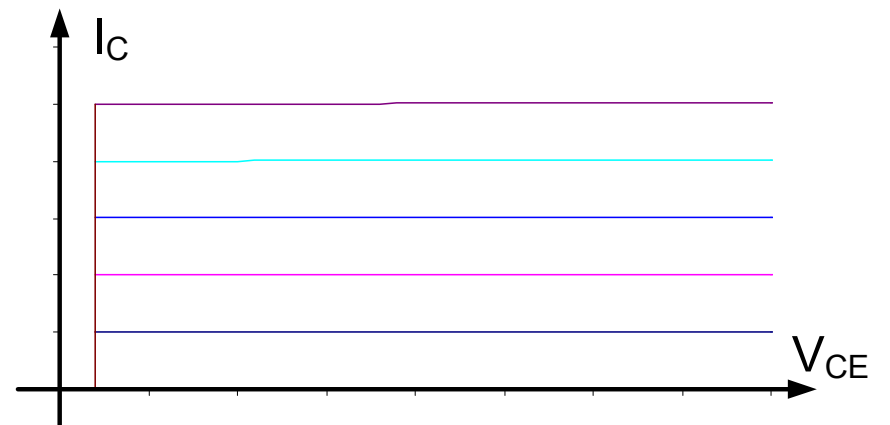
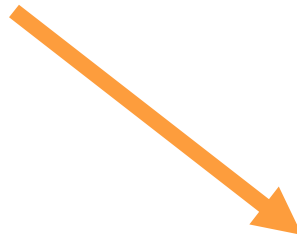
$$V_{BC} < 0$$

Cutoff

Bipolar Transistor



npn



Bipolar Transistor

Simplifier Basic Multi-Region Model

$$I_C = \beta I_B$$

$$I_B = \frac{J_S A_E}{\beta} e^{\frac{V_{BE}}{V_t}}$$

$$V_t = \frac{kT}{q}$$

$$V_{BE} > 0.4V$$

$$V_{BC} < 0$$

Forward Active

$$V_{BE} = 0.7V$$

$$V_{CE} = 0.2V$$

$$I_C < \beta I_B$$

Saturation

$$I_C = I_B = 0$$

$$V_{BE} < 0$$

$$V_{BC} < 0$$

Cutoff

End of Lecture 31